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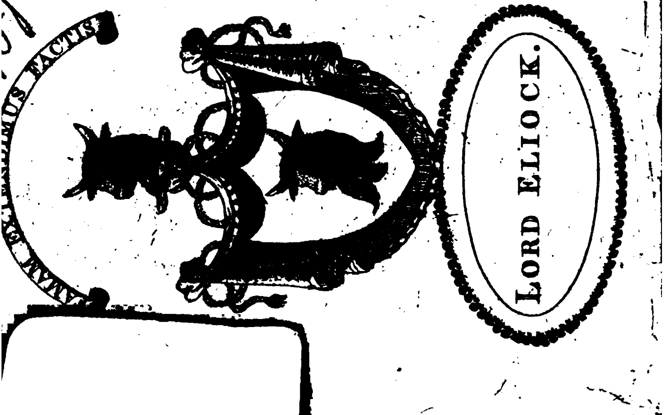

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A SHORT

TREATISE

James OF THE Veitch

General LAWS

23. OF June

MOTION

Anno. and centripetal 172

FORCES:

WHEREIN,

By the by, Mr. GORDON's Remarks on the *Newtonian* Philosophy are, in a few *Corollaries* and *Scholies*, clearly confuted.

By GEORGE PIRRIE, M. A.

EDINBURGH,
Printed for the AUTHOR, by WILLIAM ADAMS *Junior*. MDCCXX.

TO

The Right Honourable,

FRANCIS

Lord *Napier* of *Merchiston.*

My LORD,

THERE is no moral Principle more certain, plain, and incontrovertible than this, that *it is a God-like Thing to do Good*

to Mankind; of which divine Quality your Lordſhip's noble Anceſtors are honourable Examples and Inſtances.

WHEN I reflect on the great Services done to the learned World, by the moſt elaborate Performances of that very learned and worthy Gentleman *John Napier* Baron of *Merchiſton*, whoſe Praiſes, for his moſt uſeful and wonderful Invention of the *Logarithms*, beſides his other Improvements in ſolid Knowledge, deſerve to be celebrated to all Ages; I cannot but think my ſelf obliged to commemorate the Merits of ſo great a Man, and beg this ſmall

Eſſay

Eſſay may obtain the Favour and Patronage of your Lordſhip his preſent lineal Repreſentative: And I am the more encouraged to ſhelter this ſhort Treatiſe under your Lordſhip's Protection, that I am aſſur'd of your Goodneſs, and that your Mind diſcovers noble Endowments ſuperior to your Years.

YOUR Lordſhip's Family has not only adorn'd our Country with Men of eminent Learning and Knowledge in its Laws and Conſtitution, ſo that they were juſtly thought worthy to be Senators in our College of Juſtice; but has likewiſe afforded us noble Examples of true For-

titude, real Honour, and the moſt diſtinguiſh'd Loyalty, by boldly eſpouſing the Rights and Privileges of their King and Country, when groning under the Preſſure of the worſt of Times: Particularly that worthy Patriot, *Archibald* Lord *Napier*, ſuffered Impriſonment, and the greateſt Hardſhips, for cloſely adhering to the Intereſt of his Sovereign King *Charles* I. of bleſſed Memory, and endeavouring with a true Chriſtian Courage to ſupport his ſinking Country: And his Son *Archibald* Lord *Napier* (in no reſpect inferior to ſo great a Father) was forc'd at the end of an unſucceſsful War,

to retire to *Holland*, to ſhelter himſelf from the cruel Violence of the then prevailing Powers.

YOUR Lordſhip's Father, the worthy and honourable Sir *William Scot* of *Thirlſtane*, Baronet, is ſo well known to be a Gentleman of ſuch Probity, Love to his Country, polite Learning, and complete Accompliſhments, and to have taken ſuch extraordinary Care of your Lordſhip's Education; that his Memory will be always dear to all that have had the Happineſs of his Acquaintance.

MY Lord, after what I have ſaid of ſo great Examples, and

ſo worthy of Imitation, I ſhall only add my hearty Wiſhes you may not only equal, but (if poſſible) even exceed them; which cannot fail to render you the Favourite of Heaven, and the Darling of Mankind. I am with the utmoſt Reſpect,

My LORD,

Your Lordſhip's

moſt obedient and very

humble Servant,

GEORGE PIRRIE.

THE

PREFACE.

THE *ensuing Treatise is design'd to be a short, plain, and clear Introduction to the* Laws *of* Motion *and* centripetal Forces *for the Benefit of Youth, and my own private Use in Teaching. It is also partly intended for a Confutation of a small Book published last Year, which has lately receiv'd a second Edition, Intituled,* Remarks upon the Newtonian Philosophy, *by* George Gordon. *What this Gentleman the Author, or his Character in the World was, I could never certainly learn, till the second Edition of his Book came*

came abroad with a large Preface, which the first entirely wanted: And the Truth is, I was apt to think with others, that the Name was counterfeit, though now I find the contrary. In his Preface he complains heavily of very hard Usage from a Set of Men, whose Interest (he says) it is to support the Newtonian *Philosophy, and who have industriously endeavour'd to suppress his Book, and hinder every Body from buying or reading it, by decrying it in all Companies, and vilifying himself as an enterprizing Fool. And this, he thinks, they have done, because they are not able to answer his Arguments: For since Mr.* Gordon *has with no small Assurance and Confidence written his Book against the* Newtonian *System, it seems he is really perswaded, that it is impossible to confute what he has advanced to overthrow that System; and is therefore in his Preface (which, by the way, is not very civil to some learn'd Men) extremely pressing for an Answer. His Attempt is bold, and his Book manag'd with too little Respect towards two so great Men as Sir Is.*

Newton

Newton *and* Dr. Gregory; *therefore the Arguments he brings against them, and the incomparably best System of natural Philosophy that ever the World was blest with, had need be very strong and conclusive; and yet, in my Judgment, they are only the most cunning and subtil Piece of Sophistry that ever was formed on a Physico-mathematical Subject.* His *Way of reasoning does indeed, in several Places of his Book, carry a very specious Shew and Appearance of perfect Demonstration; and though stark naught at the* Bottom, *I doubt not but it has amused many, and even misled some into his* Mistakes. *I have therefore thought it proper to take his Work so far to task, as plainly to lay open the Weakness, Cheat, and Fallacy of its main Strength in a few* Scholies *and* Corollaries *of the following Tract, not reckoning it worth while to give a more large and direct Answer.*

Mr. Gordon, *by the Title Page of his Book, was obliged to shew the Fallacies of Sir* H. Newton's *and* Dr. Gregory's *mathematical Demonstrations, which he has* quite

quite omitted to do. Had he shown the Sophistry but of some few of them, I dare venture to say, nay positively to affirm, that this would have done him vastly more Service than all his fine Demonstrations; and would have made his Book pass in the World, and procur'd its Sale, in Spite of all the Opposition it could have met with from any Party whatsoever. If he thinks fit, he may attempt this in a third Edition; and if he do any Thing this way to Purpose, I cannot doubt but he will be perswaded, that I have hinted a Method to him, effectually to ruine the Reputation of the Newtonian *Philosophy, beyond any he has yet tried. But this, I believe, he will find to be the hardest Task he ever undertook in his Life.*

I do not pretend that much of the ensuing Work is new except the Method and Order: But (yet besides the foresaid Scholies *and* Corollaries, *and some Things that, I think, I have set in a better Light than any has done before) some small Part is my own, and perfectly new, as particularly* Prop. 28, 29, 30, 31, 34; *wherein, I hope*

hope, I have clearly demonstrated the Possibility that a Body may, by a projectile and a centripetal Force conjoined, move in a Circle, or any other Curve that is concave to the Center of the centripetal Force: Which Possibility has ever hitherto been supposed by all, and contradicted by none I know of before Mr. Gordon, *but never plainly demonstrated by any. At the End of this Tract, I have given a full and clear Demonstration of a short Way of arguing, often used by the incomparable* Newton *and his Followers, which, I believe, is new, and no where else to be found. I submit the whole to the Judgment of the candid Reader, hoping that he will easily pardon or overlook any light Slips and Oversights he may chance to meet with.*

A

A SHORT TREATISE OF The General Laws of Motion, and centripetal Forces.

PART I.

Of the Laws of Motion,

DEFINITIONS.

1. ANY Portion or Quantity of Matter or material Substance, is called *a Physical* or *Natural Body*; and the Space contained by its Surface, is called its *Magnitude* or *Bulk*.

2. *The Motion of a Body* is its successive Change of Place.

3. *Celerity* or *Velocity* is an Affection of Motion, whereby a Body runs through a certain Space in a certain Time.

4. *The Direction* of a Body's Motion is the straight Course, or Path, in which the Body tends.

5. *An equable* or *uniform Motion* is that, whose Celerity is neither increased nor diminished, but still continues the same.

6. *An accelerated Motion* is that, whose Velocity is still increasing.

7. *A retarded Motion* is that, whose Velocity is still decreasing.

8. *An equably* or *uniformly accelerated Motion* is that, to which in equal Times equal Degrees of Celerity are continually added.

9. *An equably retarded Motion* is that, whose Velocity in equal Times is always equally diminished.

10. The Force, Power, or Quantity of a Body's Motion, whereby it is able to produce such or such an Effect, is called its *Moment*; and so is the Force or Power of a Body, whereby it has a con-

stant

ſtant Tendency to move, often called its *Moment*, though it be not in actual Motion.

Let it here be noted, that the Words, *Moment* and *Motion*, applied to a moved Body, are commonly taken in the ſame Senſe.

11. That which reſiſts, diminiſhes, or deſtroys Motion, is called *an Impediment*.

12. *Gravitation* or *Gravity* is that Force upon Bodies, whereby they are made to move or tend to the Center of the Earth.

13. *Centripetal Force* is that, whereby a Body is conſtantly urged, or made to tend to a certain Point as a Center.

Whence 'tis plain, that Gravity is a certain Sort of Centripetal Force.

14. *A regular Centripetal Force* is that, which always acts by one conſtant Rule or Law.

15. *A Centrifugal Force* is that, whereby a Body is continually urged from a certain Point.

16. *The Center of Gravity of a Body* is that Point thereof, by which if the

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so the Body D to the Bodies A and B together; then that Point E will be called *the common Center of Gravity* of those three Bodies A, B, D.

After the same Manner, may the common Center of Gravity of four or more Bodies be defined.

19. One Body is said *to hit* or *strike directly* against another, when the right Line in which the striking Body's Center of Gravity moves, passing through the Point in which the two Bodies touch one another when they meet, is perpendicular to the Surface of the Body that is struck: But *to hit or strike obliquely*, when the foresaid Line is oblique (and not perpendicular) to the Surface of the Body that is struck.

20. *The relative Velocity* of two Bodies is the Velocity, whereby the said Bodies approach to, or recede from one another: Which is the Sum of the Velocities, when the Bodies move towards contrary Parts; and the Difference, when towards the same Part.

So when two Bodies A and B (*Fig.* 4.) move both towards the ſame Part E, the Body A with a greater Velocity, and the Body B with a leſs; the Exceſs of the Velocity of A above the Velocity of B, is the Velocity whereby A and B approach together, and conſequently is the relative Velocity of the Bodies A and B. If A move towards B at Reſt, the relative Velocity of A and B is the ſame with the Velocity of A. If the Bodies A and B move towards contrary Parts, *viz.* A towards E, and B towards D, or A towards D, and B towards E; the relative Velocity whereby A and B approach to, or recede from one another, is the Sum of the Velocities of A and B.

21. In like Manner, *the relative Motion* or *Moment* of two Bodies, is the Motion whereby the ſaid Bodies approach to or recede from one another: Which is the Sum of the Moments or Motions, when the Bodies move towards contrary Parts; and the Difference, when towards the ſame Part.

22. *A*

22. *A perfectly hard Body* is that, which yields not in the least to a Stroke, but keeps its Figure unaltered.

23. *A soft Body* is that, which changes its former Figure by a Stroke, and never of it self recovers it.

24. *An elastick Body* is that, which for some Time yields to a Stroke, but yet at last restores itself to its former Figure, at least nearly.

25. *Elasticity* or *elastick Force* is that Force, whereby a Body deprived of its former Figure, restores itself to the same Figure again.

26. *A perfectly elastick Body* is that, which restores itself to its former Figure, with a Force equal to that by which it was compress'd, and lost its Figure.

27. *Homogeneous Bodies* are Bodies of the very same Nature, Make, and Contexture of Parts; as two Pieces of Gold, or two Pieces of Lead.

28. *A void, free, or empty Space*, is a Space that is void of all Matter or material Substance.

29. If a Body A (*Fig.* 5.) moving in the right Line A B, strike against a Plane D F, and after the Stroke be reflected in the right Line B C; the Angle A B D is called *the Angle of Incidence*, and C B F *the Angle of Reflection*: But most commonly the Angle A B E (the right Line B E being perpendicular to the Plane D F) is called *the Angle of Incidence*, and C B E *the Angle of Reflection*.

30. *The absolute Quantity of a centripetal Force* is its Measure greater or less, according to the Efficacy of the Cause that propagates it around from the Center.

So the magnetical Virtue is greater in one Magnet or Load-Stone, and less in another.

31. *The accelerating Quantity of a centripetal Force*, is the Measure of the Velocity that the said Force generates in a given Time.

So the Virtue of the same Magnet is greater at a less Distance from it, and less at a greater Distance. Also at the same Distance from the Earth (the Resistence of the Air being removed) all

Bodies

Bodies descend with the same Degree of Velocity, and so their accelerating Forces are equal; but at unequal Distances they descend with unequal Velocities, and so their accelerating Forces are unequal.

MAXIMS.

1. EVERY Body or Piece of Matter will persevere in its State of Rest, or uniform Motion directly forwards; unless it be compell'd to change that State by some Force impress'd upon it.

2. Effects are proportional to their sole and adequate Causes.

So if two Forces, severally impress'd upon the same Body, be the adequate and complete Causes of two several Motions, the said Motions will be as the said Forces.

3. Equal Quantities of Matter or equal Bodies, carried with the same Velocity, have equal Moments or Quantities of Motion.

4. Equal

4. Equal and directly contrary Forces destroy one another.

5. Unequal and contrary Forces produce a Motion, that is equivalent to the Difference of the said Forces.

6. A motion that is produced by perfectly conspiring Forces (*that is*, such Forces as act according to the same Direction, and tend the very same Way) is equivalent to the Sum of the said Forces.

7. Homogeneous Bodies or their Quantities of Matter, are as the Bulks of the said Bodies.

8. If two Bodies strike directly the one against the other, the compressive Force or Magnitude of the Stroke arises from, and is equivalent to the Difference or Sum of the Moments, according as the Bodies move towards the same or contrary Parts, *that is*, to the relative Motion or Moment.

9. Action and Reaction are always contrary and equal: Or, the Actions of two Bodies upon one another are always equal, and have contrary Directions.

So if one Body press another, that first Body is equally repressed by this second; and the Directions of the Pressures are towards contrary Parts; and if one Body striking against another, by its Force make an alteration in this other Body's Motion, then that first Body's Motion will undergo an equal Alteration towards the contrary part, by Reason of the Equality of their mutual Pression. So if the Body A (*Fig.* 6.) strike directly against the Body B either at Rest, or moving more slowly towards E, the Body A will lose a Part of its Motion towards E, and the Body B will gain as much, *that is*, B will be urged by A towards E, and A will be just as much repressed or urged the contrary Way towards D by B. In like Manner, if A and B move towards one another, *viz.* A towards E, and B towards D; when they meet, they will be equally urged and pressed by one another towards contrary Parts. *Lastly*, if two Bodies A and B attract one another in Proportion to their Quantities of Matter, the Moments in both

will

will be equal; for if A be double of B, A will have double the Influence upon B that B has upon A; and consequently B will move towards A with double the Celerity of A towards B, and the Bodies A and B will be reciprocally proportional to their Celerities. Whence (as will be proved in 6 Cap. 5 Prop.) the Moments of the Bodies will be equal; and in like Manner, in any other Proportion of A to B.

10. Physical or natural Causes are not to be multiplied without good Reason.

So if one Cause will produce a certain Effect, we are not to allow of two for that Effect: If two Causes will do as well as three, we are not to allow of three, and so forth. The Reason of this Maxim is, because Nature proceeds after the simplest Method; for surely the Greatness, Wisdom, and Glory of God is more conspicuous in producing great and wonderful Effects by simple than by manifold Means.

PROP.

PROPOSITION I.

Theor. Fig. 7.

In equable and uniform Motions of Bodies, if the Times be the ſame or equal; the Lengths or Spaces run through will be proportional to the Celerities.

Let a Body in a given Time run through the Space A B with a Celerity represented by C; and in the ſame or an equal Time, let the ſame, or any other Body run through the Space D E, with a Celerity repreſented by *c*. I ſay the Line A B will be to the Line D E (both being run through with uniform Motions) as the Celerity C is to the Celerity *c*.

For if the Celerity C be double of the Celerity *c*, then the Space A B run through with the Celerity C, will be double of the Space D E run through in the ſame Time with the Celerity *c*: And if C be triple of *c*, then A B will be triple of D E: Alſo, if C be half of *c*,

then will A B be half of D E. And universally, whatever Proportion C bears to *c*, the same Proportion does A B bear to D E. W. W. D.

Corollaries.

1. HENCE, if the Times of Motion be unequal, the Spaces run through A B, D E will not be proportional to the Celerities C, *c*.

If this be denied, suppose there be as A B : D E : : C : *c*. Then since (by *hyp.*) the Times wherein the Spaces A B and D E are run through, are unequal: Let us suppose A B to be run through in a greater Time than D E, and some Part of A B, as A F, will be run through in the same Time with D E : Wherefore (by 1. *Prop.*) there will be as A F : D E : : C : *c*. But, as before, there is as A B : D E : : C : *c*. Whence as A F : D E : : A B : D E; and consequently A F = A B. W. I. A.

2. If the Spaces run through, be proportional to the Celerities, the Times of Motion will be the same or equal.

For

For if the Times be said to be unequal; then (by 1. *Cor.*) the Spaces run thro' will not be proportional to the Celerities, contrary to *Hypoth.*

PROPOSITION II.

Theor. *Fig.* 8.

IN uniform Motions, if the Celerities be equal; the Spaces run thro' will be proportional to the Times of Motion.

Suppose a Body run through the Space S in the Time T, and another Body with equal Celerity run through the Space *s* in the Time *t*: I say there will be as S : *s* :: T : *t*.

For if T be Double of *t*, then will S be double of *s*; and if T be triple, or an half of *t*, so will S be triple, or an half of *s*. And universally, whatever Proportion T bears to *t*, the same Proportion will S bear to *s*. W. W. D.

A Corollary.

HENCE, if the Times be as the Spaces, the Celerities will be equal.

This may be inferr'd from 2 *Prop.* after the ſame Manner as 2 *Cor.* 1 *Prop.* was inferred from 1 *Prop.* For we may eaſily prove, that, if the Velocities be unequal, the Spaces run thro' will not be proportional to the Times, which deſtroys the *Hypoth.*

PROPOSITION III.

Theor. *Fig.* 9.

IN compared Motions, if the Celerities be equal, the Moments or Quantities of Motion of the moved Bodies, will be as their Quantities of Matter or the Bodies themſelves.

Let two Bodies A and B be both carried with the ſame Celerity C: I ſay that

the Moment of A is to the Moment of B; as the Body A is to the Body B.

For if the Body A be double of the Body B, the Body A may be divided into two equal Parts, which moved both with the Celerity C, have (by 3 *Max.*) equal Moments, each whereof is equal to the Moment of B moved with the Celerity C: And ſo the moment of A will be double of the Moment of B. In like Manner, if A be triple of B, may we prove the Moment of A to be triple of the Moment of B. So alſo, if A be half of B, we may prove the Moment of A to be half of the Moment of B. And univerſally, the Body A is to the Body B, as the Moment of A is to the Moment of B. W. W. D.

Corollaries.

1. FROM hence and 7 *Max.* if the Bodies be homogeneous; their Moments will be as their Bulks or Magnitudes.

 2. If

2. If the Moments be as the Bodies, the Celerities will be equal.

This follows as *Cor*, 2 *Prop*.

PROPOSITION IV.

Theor. Fig. 10.

IN compared Motions of the same or equal Bodies, the Moments will be proportional to the Velocities.

Let two equal Bodies A and B be moved, the former with the Celerity C, the latter with the Celerity *c*. I say, as the Moment of A is to the Moment of B, so is the Celerity C, to the Celerity *c*.

Suppose the Bodies A and B be impell'd by two several Forces, that cause them to move with the Celerities C and *c*. Since the Bodies A and B are equal in Matter; if the Force imprest on A be double the Force imprest on B; the Moment of A will (by 2 *Max.*) be double the Moment of B, and so will C

the

the Celerity of A be double of *c* the Celerity of B.

In like Manner, if the Force impreſt on A be half of the Force impreſt on B; the Moment of A will be half of the Moment of B, and C half of *c*. And univerſally, as the Moment of A is to the Moment of B, ſo is the Celerity C to the Celerity *c*. W. W. D.

A Corollary.

IF the Moments be as the Celerities, the Bodies will be equal.

This follows as *Cor.* 2 *Prop.*

A Lemma.

IF there be any Number of Quantities of the ſame Kind, the Proportion of the firſt to the laſt is compounded of all the intermediate Proportions. So in three Quantities A, B, C, there is $\frac{A}{C} = \frac{A}{B} \times \frac{B}{C}$: in four A, B, C, D, there is $\frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}$; and ſo forth.

This is a well known Principle in Geometry.

Proposition V.

Theor. Fig. 11.

In compared Motions, the Proportion of the Moments is compounded of the direct Proportion of the Bodies and their Celerities.

Suppose two Bodies B and b be carried, the former with the Celerity C, the latter with the Celerity c; and let M denote the Moment of B, and m that of b. I say there will be $\frac{M}{m}=\frac{B}{b}\times\frac{C}{c}$.

Let G denote a third Body equal in Matter to the Body B; but suppose it moved with the Celerity c of the Body b; and let n denote its Moment. Now (by preced: *Lemma*) $\frac{M}{m}$ is $=\frac{M}{n}\times\frac{n}{m}$: but (by 4 *Prop.*) $\frac{M}{n}$ is $=\frac{C}{c}$, and (by 3 *Prop.*) $\frac{n}{m}$ is $=\frac{G}{b}=\frac{B}{b}$. Therefore is $\frac{M}{m}=\frac{B}{b}\times\frac{C}{c}$. W. W. D.

Corol-

Corollaries.

1. HENCE, the Proportion of the Celerities, is compounded of the direct Proportion of the Moments and the reciprocal Proportion of the Bodies: [*That is*, $\frac{C}{c}$ is $= \frac{M}{m} \times \frac{b}{B}$.

For (by 5 *Prop.*) $\frac{M}{m}$ is $= \frac{B}{b} \times \frac{C}{c}$: wherefore dividing both Sides by $\frac{B}{b}$, you will have $\frac{C}{c} = \left(\frac{M}{m} \div \frac{B}{b} = \frac{M \times b}{m \times B} = \right) \frac{M}{m} \times \frac{b}{B}$.

2. The Proportion of the Bodies, is compounded of the direct Proportion of the Moments and the reciprocal Proportion of the Celerities: *That is*, $\frac{B}{b}$ is $= \frac{M}{m} \times \frac{c}{C}$.

For (by 5 *Prop.*) $\frac{M}{m}$ is $= \frac{B}{b} \times \frac{C}{c}$: whence by dividing by $\frac{C}{c}$, there is $\frac{B}{b} = \left(\frac{M}{m} \div \frac{C}{c} = \frac{M \times c}{m \times C} = \right) \frac{M}{m} \times \frac{c}{C}$.

3. If the Bodies be homogeneous; the Proportion of their Moments, will be

be compounded of the direct Proportion of their Bulks and Celerities. The Proportion of the Celerities, will be compounded of the direct Proportion of the Moments, and the reciprocal Proportion of the Bulks: And the Proportion of the Bulks, of the direct Proportion of the Moments, and the reciprocal Proportion of the Celerities. As is evident from 5 *Prop.* and 1 and 2 *Cor.* and 7 *Max.*

4. The Moments M, *m* of two Bodies B, *b*, are as the Rectangles or Products B $\times$ C, *b* $\times$ *c* of the Bodies multiplied into their Celerities C, *c*: And if the Bodies be homogeneous, as the Products of their Bulks into their Celerities: As is evident from 5 *Prop.* Hence,

5. The Moment of any Body may be considered, as the Product of the said Body multiplied into its Celerity.

6. If the Moments of two Bodies be equal; the Bodies will be reciprocally proportional to their Celerities. And on the contrary.

For if M be = *m*: Since (by 4 *Cor.*) as is M : *m* :: B $\times$ C : *b* $\times$ *c*, then is B $\times$ C = *b* $\times$ *c*,

$b \times c$, and consequently as $B : b :: c : C$. And on the contrary, if there be as $B : b :: c : C$, then is $B \times C = b \times c$, and consequently (by 4 *Cor.*) $M = m$.

7. The Velocities of Bodies are as the Moments applied to the Bodies, *that is*, as the Quotes resulting from the Moments divided by the Bodies.

For (by 1 *Cor.*) $\frac{C}{c}$ is $= \frac{M}{m} \times \frac{b}{B} = \frac{M}{B} \div \frac{m}{b}$: and consequently as is $C : c :: \frac{M}{B} : \frac{m}{b}$. Hence,

8. The Velocity of a Body may be considered as the Moment applied to, or divided by the Body.

9. Bodies are as their Moments applied to their Celerities.

For (by 2 *Cor.*) $\frac{B}{b}$ is $= \frac{M}{m} \times \frac{c}{C} = \frac{M}{C} \div \frac{m}{c}$: and consequently as $B : b :: \frac{M}{C} : \frac{m}{c}$. Hence,

10. A Body may be considered as its Moment applied to its Velocity.

PROP.

PROPOSITION VI.

Theor. *Fig.* 12.

IN uniform Motions, the Spaces run through are in a compound Proportion of the Times and Celerities.

Let the Line S be a Space run thro' with the Celerity C in the Time T, and the Line s another Space run thro' with the Celerity c in the Time t: I say there will be $\frac{S}{s} = \frac{T}{t} \times \frac{C}{c}$.

Let F be a Space run thro' with the Celerity C in the Time T. Then (by *Lem.* 5 *Prop.*) is $\frac{S}{s} = \frac{S}{F} \times \frac{F}{s}$: but because S and F are run thro' in the same Time T, therefore (by 1 *Prop.*) is $\frac{S}{F} = \frac{C}{c}$. Again, because F and s are run thro' with the same Celerity c, therefore (by 2 *Prop.*) is $\frac{F}{s} = \frac{T}{t}$; whence $\frac{S}{s}$ is $= \frac{C}{c} \times \frac{T}{t}$. W. W. D.

Corol-

Corollaries.

1. The Proportion of the Times, is compounded of the direct Proportion of the Spaces, and the reciprocal Proportion of the Velocities; *that is*, $\frac{T}{t}$ is $= \frac{S}{s} \times \frac{c}{C}$.

For (by 6 *Prop.*) $\frac{S}{s}$ is $= \frac{T}{t} \times \frac{C}{c}$. Therefore, by dividing by $\frac{C}{c}$, is $\frac{T}{t} = \left(\frac{S}{s} \div \frac{C}{c} = \frac{S \times c}{s \times C} = \right) \frac{S}{s} \times \frac{c}{C}$.

2. The Celerities are in the direct Proportion of the Spaces, and the reciprocal Proportion of the Times; *that is*, $\frac{C}{c}$ is $= \frac{S}{s} \times \frac{t}{T}$.

This is proved from 6 *Prop.* after the same Manner as 1 *Cor.*

3. The spaces run thro' s, *s*, are as the Products c x T, *c* x *t* of the Celerities and Times; as is evident from 6 *Prop.* And so any Space run thro' may be consider'd as the Product of the Celerity into the Time.

4. If the Spaces be equal, the Celerities will be reciprocally proportional to the Times; and on the contrary.

For if S be = s; then (by 3 *Cor.*) is $C \times T = c \times t$, and consequently as $C : c :: t : T$. And if there be as $C : c :: t : T$, then is $C \times T = c \times t$, and so $S = s$.

5. The Time is as the Space applied to the Celerity.

For (by 1 *Cor.*) $\frac{T}{t}$ is $= \frac{S}{s} \times \frac{c}{C} = \frac{S}{C} \div \frac{s}{c}$: Whence as $T : t :: \frac{S}{C} : \frac{s}{c}$.

6. The Celerity is as the Space applied to the Time.

For (by 2 *Cor.*) $\frac{C}{c}$ is $= \frac{S}{s} \times \frac{t}{T} = \frac{S}{T} \div \frac{s}{t}$: Whence as $C : c :: \frac{S}{T} : \frac{s}{t}$.

PROPOSITION VII.

Theor.

I*N uniform Motions, the Moments are as the Products of the Bodies and Spaces applied to the Times; that is, (the Bodies, Moments, Spaces, and Times*

Times being respectively denoted by B, *b*; M, *m*; S, *s*; T, *t*; *as before*) *there is*, as M : *m* :: $\frac{B \times S}{T}$: $\frac{b \times s}{t}$.

For (by 4 *Cor.* 5 *Prop.*) $\frac{M}{m}$ is $= \frac{B \times C}{b \times c} = \frac{B}{b} \times \frac{C}{c}$. But (by 6 *Cor.* 6 *Prop.*) as C : *c* :: $\frac{S}{T}$: $\frac{s}{t}$, and so $\frac{C}{c} = \frac{S}{T} \div \frac{s}{t} = \frac{S \times t}{s \times T}$: Therefore is $\frac{M}{m} = \frac{B}{b} \times \frac{S \times t}{s \times T} = \frac{BS}{bs} \times \frac{t}{T} = \frac{BS}{T} \div \frac{bs}{t}$: Whence as M : *m* :: $\frac{BS}{T}$: $\frac{bs}{t}$. W. W. D.

A Schol.

THE same Thing may be more briefly expressed thus; *the Moment is as the Product of the Body and Space applied to the Time*, that is, M is as $\frac{BS}{T}$; and very compendiously demonstrated thus.

By 5 *Cor.* 5 *Prop.* M is as BC: But (by 6 *Cor.* 6 *Prop.*) C is as $\frac{S}{T}$. Therefore

fore ($\frac{S}{T}$ being put for c) M is as B × $\frac{S}{T}$ or $\frac{BS}{T}$.

A Corollary.

Hence, if the Times be equal, the Moments will be as the Products of the Bodies and Spaces.

PROPOSITION VIII.

Theor. Fig. 13.

If there be, in a void Space, two soft or perfectly hard Bodies, free from the Action and Influence of all other Bodies; and the one strike directly against the other, whether that upon which the Stroke is made be at Rest, or moves more slowly towards the same Part, or lastly towards the contrary Part with a less Motion or Moment: After the Stroke, they will both move close together, with one and the same Degree of Velocity, towards that Part whither the striking Body tended.

Let

Let the Body B moving towards E strike directly upon the Body b, either at Rest, or moving towards E with less Celerity than that of B, or moving the contrary Way towards D with a smaller Quantity of Motion. Then the Body b when struck by the Body B, will move towards E. But b cannot move slower than B, by reason of B following it; nor can b move faster than B, there being (by *Hyp.*) no Elasticity, nor any other Cause to separate them when met. Therefore the Bodies B and b, after Collision, will both move close together towards E, with the same Degree of Velocity.

Proposition IX.

Theor.

IF, *in a void Space, two perfectly elastick Bodies (free from the Influence of all other Bodies) strike directly the one against the other; their relative Velocity after the Stroke, will be equal to their re-*

lative Velocity before the Stroke: And consequently the Bodies, after Collision, will recede from one another with the same Velocity wherewith they approach'd before.

For in any two Bodies when they meet, the compressive Force is, (by 8 *Max.*) equivalent to the relative Motion before the Stroke: But in perfectly elastick Bodies, the compressive Force is (by 26 *Def.*) equivalent to the Elasticity or restitutive Force. Therefore the relative Motion before the Stroke, is equivalent to the restitutive Force after the Stroke: And this relative Motion is the sole Cause of the relative Velocity before the Stroke. Now, if there was no restitutive Force, the Bodies, after the Stroke, would (by 8 *Prop.*) move close together with the same Velocity, without any relative Velocity: But the restitutive Force makes them separate, and recede from one another after the Stroke, and so creates a new relative Velocity, whereof the said restitutive Force is the sole Cause. Therefore the relative Motion

before the Stroke, and the restitutive Force after the Stroke, being, as before, equivalent, and the sole Causes of the relative Velocity before and after the Stroke, it is evident (from 2 Max.) that the said relative Velocities are equal. And consequently, the Bodies after Collision, will recede from one another just as fast as they approached before. W. W. D.

Proposition X.

Theor. Fig. 13.

If two Bodies, moving (in a free Space) either towards the same or contrary Parts, strike directly the one against the other; the Sum of the Motions or Moments towards one and the same Part, will be the same, after the Bodies strike, that it was before.

Let B and *b* be two moving Bodies, and let C be the Celerity of B, and *c* that of *b*; also let *m* denote the Moment lost to B, and communicated by B to *b* by the Stroke.

1. *Case*.

1. *Case.* Suppose the Bodies B, b, both move towards the same Part E, before the Stroke. Then (by 5 *Cor.* 5 *Prop.*) B C is the Moment or Motion of B towards E, before the Stroke, and bc the Motion of b towards E; whose Sum is B C + bc: But after the Stroke, the Motion of B towards E is B C − m, and that of b towards E is bc + m; whose Sum is B C + bc, as before.

2. *Case.* Suppose the Bodies B, b, before the Stroke, move towards contrary Parts, *viz.* B towards E, and b towards D: Then (by 5 *Cor.* 5 *Prop.*) B C is the Moment or Motion of B towards E, and (bc the Motion of b towards D, and so) − bc the Motion of b towards E; *that is*, B C − bc is the Sum of the Motions towards E before the Stroke. But after the Stroke, the Motion of B towards E, is B C − m, and that of b towards E is − bc + m; whose Sum is B C − bc, as before.

A Scho-

A Scholy.

IF the Body b be at Rest before the Stroke, *that is*, if c be $=$ o, and consequently $bc =$ o; then the Motion of the Body B alone towards E, before the Stroke, is the whole Motion of both Bodies B and b towards E after the Stroke. Because, in that Case, $BC \pm bc$ is $= BC - m \pm bc + m = BC$.

Proposition XI.

Probl. Fig. 13.

TO determine the Celerities and Motions of soft and perfectly hard Bodies, after they strike directly one against another.

That two such Bodies, after the Stroke, will move with one common Velocity, is evident from 8 *Prop.* to determine which, suppose the Body B move with a greater Motion, and the Body b with a less, towards the same or contrary

Parts;

Parts; and let C denote the Celerity of B, before the Stroke, and c that of b. Now if B and b move towards the same Part E, the Sum of their Motions towards that Part, before they strike, will be $BC + bc$; but if towards contrary Parts, *viz.* B towards E, and b towards D, the Sum of the Motions towards E, before the Stroke, will be $BC - bc$. But (by 10 *Prop.*) the Sum of the Motions towards the same Part, is the same before and after the Stroke. Therefore the whole Motion of the Bodies towards E, after the Stroke, is $BC + bc$ or $BC - bc$, according as they tended towards the same or contrary Parts before the Stroke. Therefore (by 8 *Cor.* 5 *Prop.*) $\frac{BC + bc}{B + b}$, or $\frac{BC - bc}{B + b}$ will give the common Velocity towards E, after the Stroke.

If the Body b be at Rest before the Stroke, *that is*, if c be $= 0$, the common Velocity of the Bodies, after the Stroke, will be $\frac{BC}{B + b}$.

If

If the Bodies B and b move towards contrary Parts, with equal Motions or Moments; then will $\frac{BC - bc}{B + b}$ be $= 0$, *that is*, their common Celerity, after the Stroke, will be nothing, and so they will both rest.

Therefore, if the Bodies B, b, be given, and also their Celerities C, c, before the Stroke; the common Celerity, after the Stroke, will easily be found. For Instance, suppose B $= 3$, $b = 2$, C $= 7$, $c = 5$; and suppose the Bodies move both towards E, before the Stroke; then is $\frac{BC + bc}{B + b} = \frac{31}{5} = 6\frac{1}{5}$ the Degrees (after the Stroke) of the common Velocity towards E. If B, b move contrary Ways before the Stroke; then is $\frac{BC - bc}{B + b} = 2\frac{1}{5}$ the common Velocity (after the Stroke) yet towards E, because BC is more than bc.

The Moment of each Body, after the Stroke is had, by multiplying each into the common Velocity; as is evident from 5 *Cor.* 5 *Prop.*

A Lem-

A Lemma. Fig. 14.

IF *two Bodies* B, *b*, *move both towards the same Part* E, B *with a slower Motion, and b with a swifter; or the Body b towards* E, *and* B *towards the contrary Part* D, *with equal or unequal Motions: Then in both Cases, the relative Celerity of these Bodies added to the Celerity of the Body* B, *will give the Celerity of the Body b.*

The first Case is evident; because the relative Celerity in that Case, is (by 20 *Def.*) the Difference of the two simple Celerities; and the Difference of any two Quantities added to the lesser gives the greater.

In the second Case let C denote the Celerity of the Body B, and *c* that of the Body *b*; then (by 20 *Def.*) C + *c* is their relative Celerity. Now if we consider the Celerity *c*, as a positive Quantity, the Celerity C being the direct contrary Way will be a negative one,

and so c + c added to − c gives c the Celerity of the Body b.

PROPOSITION XII.

Probl. Fig. 14.

TO determine the Velocities of perfectly elastick Bodies, after they strike directly one against another.

1. Suppose two perfectly elastick Bodies B, b, move towards the same Part E before they strike, B with the Celerity c, and b with the Celerity c: Whence the relative Celerity, before the Stroke, will (by 20 *Def.*) be c − c. Therefore, since (by 9 *Prop.*) c − c is also the relative Celerity after the Stroke, as well as before; if we put x for the Celerity of B, and z for that of b, after the Stroke, and leave x undetermined to the Sign + or −, (because though b, after the Stroke, must certainly move towards E, yet B after the Stroke may move either towards E or D) we will have x + c − c = z, by pre-

ceed. Lem. For Inſtance, if x be $= 3$; aud $z = 5$; then $C - c$ the relative Velocity, after the Stroke, is either 2 or 8, according as the Body B, after the Stroke, moves towards E or D; and $+ 3 + 2$ is $= 5$, alſo $- 3 + 8 = 5$.

Now the Motion of B towards E, after the Stroke (by 5 *Cor.* 5 *Prop.*) is B x with an undetermin'd Sign; and the Motion of b towards E, after the Stroke, is ($bz =$) $bx + bC - bc$: And ſo the Sum of theſe two Motions, or the whole Motion of B and b towards E, after the Stroke, is $Bx + bx + bC - bc$: But the whole Motion towards E, before the Stroke, is $BC + bc$. Therefore (by 10 *Prop.*) is $Bx + bx + bC - bc = BC + bc$: Whence $Bx + bx = BC + 2bc - bC$: And ſo $x = \frac{BC + 2bc - bC}{B + b}$; which Celerity x (of the Body B after the Stroke) will be either poſitive, and ſo towards E, or elſe negative, and ſo towards D, according as $BC + 2bc$ is more or leſs than bC.

Again,

Again, the Celerity of b, after the Stroke, *viz.* z is $(= x + \text{C} - c = \frac{\text{BC} + 2bc - b\text{C}}{\text{B} + b} + \text{C} - c) = \frac{2\text{BC} + bc - \text{B}c}{\text{B} + b}$.

Wherefore, if two perfectly elastick Bodies B, b, moving to the same Part E, be given, and also their Celerities C, c, before the Stroke; it will be easy to find their Celerities x, z, after the Stroke; and to what Parts they tend. For Instance, suppose $\text{B} = 3$, $b = 2$, $\text{C} = 7$, $c = 5$; then is $x \left(= \frac{\text{BC} + 2bc - b\text{C}}{\text{B} + b} = \frac{21 + 20 - 14}{3 + 2} \right) = + 5\frac{2}{5}$; *that is*, the Body B, after the Stroke, moves yet towards E (by Reason of the positive Sign) with $5\frac{2}{5}$ Degrees of Celerity. Again, z is $\left(= \frac{2\text{BC} + bc - \text{B}c}{\text{B} + b} = \frac{42 + 10 - 15}{5} \right) = + 7\frac{2}{5}$, *that is*, the Body b, after the Stroke, moves towards E with $7\frac{2}{5}$ Degrees of Celerity. And the relative Velocity, after the Stroke, $7\frac{2}{5} - 5\frac{2}{5}$ is

equal to $7 - 5$, the relative Velocity before the Stroke, as it ſhould be.

If B be $= 2$, $b = 5$, C $= 13$, $c = 3$; then $x = \left(\frac{BC + 2bc - bC}{B + b} \text{ is } =\right) - 1\frac{2}{7}$: Wherefore the Sign of $1\frac{2}{7}$ or x being negative, the Body B, after the Stroke, moves backwards towards D with the Celerity $1\frac{2}{7}$. Again, $z \left(= \frac{2BC + bc - Bc}{B + b}\right)$ is $= + 8\frac{5}{7}$: Wherefore the Body b, after the Stroke, moves towards E with the Celerity $8\frac{5}{7}$.

2. If the elaſtick Bodies, B, b, before the Stroke, move with the Celerities C, c, towards contrary Parts, *viz.* B towards E, and b towards D; their relative Celerity, before the Stroke, will be C $+ c$. Therefore (by 9 *Prop.*) C $+ c$ will alſo be their relative Celerity after the Stroke; and if we put x, z for their ſimple Celerities after the Stroke, leaving the Signs both of x and z undetermined, there will (by *preceed. Lem.*) be $x + C + c = z$, Now the Motion of B to-

towards E; after the Stroke, is Bx, and that of b towards E is ($bz =$) $bx + bC + bc$; whose Sum $Bx + bx + bC + bc$ is (by 10 *Prop.*) $= BC - bc$, the whole Motion of B and b towards E before the Stroke. Whence $Bx + bx$ is $= BC - bC - 2bc$; and x (the Celerity of B after the Stroke) $= \frac{BC - bC - 2bc}{B + b}$ positive or negative, according as B C is more or less than $bC + 2bc$.

Again, z (the Velocity of b after the Stroke) is $= x + C + c = \frac{BC - bC - 2bc}{B + b} + C + c = \frac{2BC + Bc - bc}{B + b}$.

Whence, in the Case of two perfectly elastick Bodies B, b, moving towards contrary Parts E, D; the Bodies themselves, and their Celerities C, c, before the Stroke, being given; we may easily find their Celerities x, z, after the Stroke, and to what Parts the Bodies tend. For Example, suppose $B = 3$, $b = 2$, $C = 7$, $c = 5$; then is $x = \left(\frac{BC - bC - 2bc}{B + b} = \right) - 2\frac{3}{5}$; Therefore

the Body B before the Stroke, moving towards E with 7 Degrees of Velocity, will after the Stroke move towards the contrary Part D (because of the negative Sign before $2\frac{3}{5}$) with $2\frac{3}{5}$ Degrees of Velocity. Again, z is $= \left(\frac{2BC + Bc - bc}{B + b} =\right) + 9\frac{2}{5}$; therefore the Body b, after the Stroke, will move towards E with $9\frac{2}{5}$ Degrees of Velocity.

Corollaries.

1. IF the elastick Body B move towards E before the Stroke, and the other Body b be at Rest; then is $c = 0$, and consequently $x = \frac{BC - bC}{B + b}$, and as $B + b : B - b :: C : x$. Again, since c is $= 0$, therefore is $z = \frac{2BC}{B + b}$, and as $B + b : 2B :: C : z$. Hence, if B be $= b$ at Rest, then is $x = \left(\frac{BC - BC}{2B} =\right) 0$, *that is*, B, after the Stroke, rests; and $z = \left(\frac{2BC}{2B} =\right)$ C, that

c, *that is*, *b*, after the Stroke, moves towards E with the Celerity that B had before the Stroke.

2. If the elastick Bodies B and *b* be equal, and move towards contrary Parts before the Stroke: Then is $x = \left(\frac{-2Bc}{2B} =\right) - c$; and $z = \left(\frac{+2BC}{2B} =\right) + C$; *that is*, after the Stroke, the Bodies B, *b*, will move towards contrary Parts with interchanged Velocities, *viz.* B, after the Stroke, with the Velocity of *b* before the Stroke, and *b*, after the Stroke, with the Velocity of B before the Stroke.

3. If the Bodies B, *b*, be equal, and move towards the same Part before the Stroke, *b* going before with the Velocity *c*, and B following with a greater Velocity C; then is $x = \left(\frac{+2bc}{B+b} = \frac{+2bc}{2b} =\right) + c$, and $z = \left(\frac{+2BC}{B+b} = \frac{+2BC}{2B} =\right) + C$.

4. If the Bodies B, *b*, (whether equal or unequal) move towards contrary Parts

be-

before the Stroke, and there be as B : b :: c : C; then is B C $= b\,c$, *that is*, the Moments or Motions are equal; and x $\left(= \frac{BC - bC - 2bc}{B + b} = \frac{-BC - bC}{B + b}\right)$ is $=$ $-$C, and z $\left(= \frac{2BC + Bc - bc}{B + b} = \frac{bc + Bc}{B + b}\right)$ is $= +c$.

Proposition XIII.

Theor. *Fig.* 15.

If *a Body* A *move uniformly in the right Line* A B, *whilst the said Line* A B, *moves uniformly always parallel to itself, keeping its Extremity* A *in the right Line* A C; *and the Parallelogram* A B D C *being completed, if the Velocity of the Body* A *in the Line* A B, *be to the Velocity of the Line* A B *itself, as* A B *to* A C: *Then the Body* A *by this compound Motion, will really describe the Diagonal* A D *in the same Time that it describes the Line* A B; *or that the End* A *of the Line* A B *describes the Line* A C.

When the Line A B comes to any other Situation *a b*, let *g* be the Place of the Body A; and draw *g* G parallel to A C. Now, ſince the Spaces run thro' in the ſame Time, by the Body A in the Line A B or *a b*, and by the Line A B itſelf, are A G or *a g* and A *a* or G *g*; therefore (by 1 *Prop.*) as is A G to A *a*, ſo is the Velocity of the Body A in the Line A B to the Velocity of the Line A B, *that is*, (by *Hyp.*) A B to A C. Whence the Paralellograms *a* G and C B are ſimilar: Therefore the Point *g* is (by 26. 6. *Eucl.*) in the Diagonal A D; and conſequently (ſince the Point *g* was taken at Pleaſure, *a b* being taken in any Situation parallel to A B) the Body A will always be in the Diagonal A D. And that it will deſcribe the Diagonal A D in the ſame Time that it deſcribes the Side A B, or that A B deſcribes A C, is evident; becauſe (by *Hyp.*) the Velocity of the Body A in the Line A B is to the Velocity of the Line A B as A B to A C, and conſequently (by 2 *Cor.* 1 *Prop.*) in the ſame Time that the Body A, moving

in

in A B, describes A B, in the same Time A B describes A C, and at the End of the said Time coincides with C D.

A Corollary.

HENCE it is evident, that, if a Body A be urged by two Forces together, the one acting in the Direction A B, and the other in the Direction A C; the Force acting in the Direction A B will not at all hinder the Motion (by the other Force) of the Body A towards C D parallel to A B, nor will the Force in the Direction A C hinder the Motion of the Body A towards B D parallel to A C.

PROPOSITION XIV.

Theor. Fig. 15.

IF a Body A *have two Forces imprest upon it together in different Directions; by which Forces acting separately, it would uniformly describe the Sides* A B, A C *of a Parallelogram* A B D C *in equal Times,*

or

or (by 1 Prop.) *with Velocities as* A B, A C: *I say, that the Body* A *with these two Forces united, will describe the Diagonal* A D, *in the same Time that it would describe* A B, *or* A C *with the one, or the other of the said Forces acting separately.*

Suppose the Body A could describe the Side A B, in the Time T, with a certain Force M; and in the same or equal Time, the Side A C, with another Force N: The Force N acting in the Direction A C, is (by *Cor.* 13 *Prop.*) no Impediment to the Body's Motion towards B D, arising from the Force M; the Body therefore will be carried to the Line B D, in the same Time T, whether the Force N be impreſt or not: Therefore at the End of the Time T, it will be found somewhere in the Line B D. By the same Argument, it will at the End of the Time T be found somewhere in C D; and therefore of Necessity in the Concourse D of the said Lines B D and C D; and so by the Forces M and N united, or the compound Force, moves in the Diago-

Diagonal A D. Wherefore the Velocities in A B, A D, A C, will (by 1 *Prop.*) be as the Spaces A B, A D, A C.

Corollaries.

1. THE Forces M, N, and the compound Force resulting thence; also the Motions, and Velocities arising from these three Forces respectively, by which the Sides A B, A C, and the Diagonal A D may be described by the same Body in equal Times, are proportional to, and consequently may be expounded by the said Sides and Diagonal respectively.

For (by 1 and 14 *Prop.*) the Lines A B, A C, A D are proportional to the Velocities in A B, A C, A D; and these Velocities are (by 4 *Prop.*) proportional to the Moments or Motions in A B, A C, A D; and these Motions are (by 2 *Max.*) proportional to the Forces M, N, and the compound Force, acting in the Directions A B, A C, A D. Hence,

2. If the Diagonal A D and Side A B be equal; the Velocity in A D resulting from the compound Force, will be equal to the Velocity in A B resulting from the simple Force in A B.

3. Any Force or Motion, though in itself ever so simple, may be considered as compounded of other Forces or Motions. Thus the Motion as A D (*Fig.* 16.) in the Diagonal A D of the Parallelograms C B and F E, may be resolved into the Motions as A B and A C in the Sides A B and A C, and also into the Motions as A E and A F in the Sides A E and A F. And on the contrary, two Forces as A B and A C, urging in the Directions A B and A C, are united equivalent to one single Force as AD in the Direction A D.

Proposition XV.

Theor. *Fig.* 17.

IF *a Body in* A *be urged by a Force as* A B *in the Direction* A B, *by which in a certain Time it would uniformly describe*

ſcribe the Line A B; *but really in the ſame Time deſcribes the right Line* A D: *The ſaid Body is alſo impell'd in* A, *in the Direction* A C *parallel to the right Line* B D *that joins the Points* B *and* D, *by ſome other Force, as* A C, *equal to* B D.

For if A C be not the other Force in the Direction A C, whereby the Body in A is impell'd; it will be ſome other Force as A K, in the Direction A C or A K, or as A F in a different Direction A F. Complete the Parallelograms B A C D, B A K L, B A F E, and draw the Diagonals A L, A E.

Firſt, let the other Force be A K in that Direction; then A D (by *Hyp.*) is really deſcrib'd by the Body, in the Time that A B would have been deſcrib'd by the Force A B; and A L (by 14 *Prop.*) is alſo really deſcrib'd in the Time that A B would have been deſcrib'd: Therefore A D and A L are both really deſcrib'd by the ſame Body, in one and the ſame Time. W. I. A.

Secondly, if A F be the other Force in the Direction A F, by which the Body in

in A is urged; the Diagonal A E will either be a different Line from the Diagonal A D, or that Line will fall upon this and be of different Length: Then just as before (by *Hyp.* and 14 *Prop.*) A D and A E will both really be describ'd by the Body, in one and the same Time, *viz* in the Time that A B would have been describ'd by the Force A B. W. I. A.

A Scholy. Fig. 18.

LET A B and A S be two right Lines making any Angle at A, and let a Body in A be impell'd or urged, at the same Instant of Time, by two Forces in the Directions A B and A S; then it is plain, that one of the said Forces may be so adjusted and proportioned to the other, as to make the Body move in any Direction A C between A B and A S.

PROPOSITION XVI.

Probl. *Fig.* 19.

TO determine the Directions and Celerities, after the Stroke, of Bodies striking one another obliquely.

Let two Bodies A, B, move in the right Lines AC, BC, that incline to one another; and let AC, BC express the Proportion of the Motions of these Bodies: Let the right Line ECH represent a Plane which the Bodies touch in the Point of Concourse C; to which Plane demit from A and B the Perpendiculars AE and BF, and complete the Rectangles EG, FH.

The Motion as AC of the Body A in the Direction AC, may (by 3 *Cor.* 14 *Prop.*) be resolved into other two Motions as AE and AG in the Directions AE and AG; which three Motions are (by 4 *Prop.*) as the Velocities; and consequently the Velocities are as AC, AE, AG. Therefore the Velocity as AC in the

the Direction A C, may be resolved into the Velocities as A E, A G, in the Directions A E, A G. For the same Reasons, the Velocity of the Body B as B C in the Direction B C, may be resolved into the Velocities as B F, B H in the Directions B F, B H. But A G and B H being parallel, the Velocities A G, B H, in the Directions A G, B H, do nothing to make the Bodies approach and strike, and so are not at all concerned with, nor altered by the Stroke. Therefore the Velocities, whereby the Bodies meet and strike one another, are only those that are as A E or G C, and B F or H C in the Directions G C and H C. Therefore the Bodies A and B striking directly against one another with the Velocities G C and H C, their Velocities, after the Stroke in the Line H G, may be determined by 11 *Prop.* if they be soft or perfectly hard Bodies, or by 12 *Prop.* if they be perfectly elastick

Suppose then C L was thus found to be as the Velocity, and consequently as the Force of the Body A, moving

from C towards G, after the Stroke; since, as before, the Force in the Body A, of moving in the Direction A G or E C with the Velocity A G, is not altered by the Stroke, produce E C till C M be = E C, and complete the Rectangle L M; the Body A, after the Stroke, will (as is evident from 14 *Prop.*) move in the Diagonal C N, with a Velocity as C N. In like Manner, if (by 11 or 12 *Prop.*) we find the Velocity of the Body B, after the Stroke, to be as Q C in the Line G H; its true Velocity and Direction, after the Stroke, may be determined, by making C S = F C, and completing the *Rectangle* Q S: For then the Body B, after the Stroke, will run in the Diagonal C R, with a Velocity as C R.

Example. Let two perfectly elastick spherick Bodies A, B, as 2, 3, move in the oblique Directions A C, B C, with Velocities as A C, B C, or 7, 5; and let it be required to find the Velocities and Directions of the said Bodies, after they meet and strike in C. Bisect the Angle B C A by the right Line E C M, and

and ECM will represent the Plane which the Bodies A and B touch in C. Then (the Triangles ACE, BCF being similar) there will be as AC : BC (:: 7 : 5) :: AE : BF :: GC : HC :: AG : BH :: EC : FC :: (by making CM = EC, and CS = FC) CM : CS. Let x denote the Velocity (after the Stroke) of the Body A referred to the Line HG, and z the Velocity (after the Stroke) of the Body B referred to the same Line: Then since the Bodies A and B are as 2 and 3, and the Velocities whereby they strike directly, the one against the other, are as GC and HC, or as 7 and 5; we will find (by 2 *Part* 12 *Prop.*) $x = -7'4$, and $z = +4'6$. Therefore, after the Stroke, the Bodies A and B will move towards contrary Parts, *viz.* A from C towards G, with a Velocity as 7'4 referr'd to the Line HG, and B from C towards H, with a Velocity as 4'6 referr'd to the same Line. Therefore, since CM and CS are as 7 and 5, if we make GL = 7'4, and CQ = 4'6, and complete the Rectangles LM and QS; there will

will be $CN = \sqrt{MNq + CMq} = \sqrt{54'76 + 49} = 10'18$, and $CR = \sqrt{RSq + CSq} = \sqrt{21'16 + 25} = 6'79$. Wherefore the Bodies A and B, after the Stroke, will really move in the Diagonals CN and CR, with Velocities as 10'18 and 6'79.

Proposition XVII.

Theor. Fig. 20.

A Stroke made by a Body A *upon a firm and immoveable Plane* E F, *in an oblique Direction* A C, *is to one made in a perpendicular Direction* A D, *the Body moving separately in both Directions with the same Degree of Velocity, as* A D *is to* A C, *or as the Sine of the Angle of Incidence* A C D *is to the Radius.*

The Rectangle A D C B being compleated, the Motion of the Body A, resulting from a Force as A C, in the Direction A C, is (by 3 *Cor.* 14. *Prop.*) equivalent to two other Motions in the Lines A D and A B, resulting from two other

other Forces as A D and A B. But the Force or Motion whose Direction is A B, is of no Significancy as to the Stroke upon the Plane E F; because A B being parallel to E F, the Body moving in the Direction A B, would never meet with E F. Therefore the Force by which the Body moves in A C being as A C, that Force by which it strikes upon the Plane in that oblique Direction A C, is as A D. But if the Body mov'd in the perpendicular Direction A D, with a Force as A C, and consequently with the same Velocity as it moves in A C, the Quantity of the perpendicular Stroke would be equivalent to the Force as A C, because the said Force would be wholly destroy'd by the perpendicular Stroke. Therefore the Quantity of the oblique Stroke is to the Quantity of the perpendicular Stroke, as A D is to A C, *that is*, (A C being made Radius) as the Sine of the Angle of Incidence A C D is to the Radius. W. W. D.

PRO-

PROPOSITION XVIII.

Theor. Fig. 21.

IF a perfectly-elastick Body A, *moving in the right Line* AB, *strike obliquely against a firm and immoveable Plane* HG: *After the Stroke it will be reflected by that Plane with the same Force that it came with; and will move in such a right Line* BC, *that the Angle of Reflection* CBF *will be equal to the Angle of Incidence* ABD.

Let AB represent the Force, or Motion of the Body A, in the Direction AB; then, AD being drawn perpendicular to the Plane HG, and the Rectangle DE completed, the Motion in AB is (by 3 *Cor.* 14 *Prop.*) equivalent to two Motions, in the Directions AD and AE, proportional to the Lines AD and AE. But since AE is parallel to HG, and AD perpendicular thereto; the Force by which the Body strikes against the Plane, is only that which is as AD. Make BF = DB,

D B, or A E, and complete the Rectangle E F, which will be every Way equal and ſimilar to the Rectangle D E, and conſequently B C = A B, and > C B F = > A B D. Since the Force as A E, acting in the Direction A E parallel to the Plane H G, is not diminiſhed by the Stroke, the Force A E remains in the Body A after the Stroke, to move it in the Direction B F or A E. But from the Nature of a perfectly elaſtick Body it is evident, that the Body A, ſtriking the immoveable Plane H G, in the perpendicular Direction A D or E B, will reflect with the ſame Force in the ſame Line of Direction: Therefore the Motion of the Body A, at the Point of Incidence B, is compounded of the Motions as B F and B E in the Directions B F and B E. Therefore (by 14 *Prop.*) the Body A, after the Stroke, will move in the Diagonal B C of the Rectangle E F, with a Force as B C equal to A B; and the Angle of Reflection C B F is (as before) equal to the Angle of Incidence A B D. W. W. D.

PRO-

PROPOSITION XIX.

Theor.

ALL *Bodies near the Surface of the Earth, gravitate* (*in a free Space*) *in Proportion to their Quantity of Matter*; that is, *their Weights are proportional to the Bodies themselves*.

For it is known by many Experiments, that all Bodies near the Earth's Surface, falling perpendicularly by the Force of Gravity, in a free Space, descend equal Spaces in equal Times: And therefore, at the End of any Time given, they acquire equal Velocities. Therefore the Motions or Moments acquired at the End of the said Times, being (by 4 *Cor.* 5 *Prop.*) as the Bodies multiplied into their equal Velocities, are as the Bodies themselves. But the Forces that generate these Moments, *that is*, the Gravitations or Weights of the Bodies, are (by 2 *Max.*) proportional to the said Moments. Therefore the

Weights are proportional to the Bodies. W. W. D.

Corol. Hence, a Body may be confidered as its Weight.

Proposition XX.

Theor.

THE Motion of a defcending Body near the Surface of the Earth, falling from Reft, in a free Space, by the Force of its Gravity, is an equably accelerated Motion.

For the Gravity or Weight of a Body near the Earth, is not fenfibly altered by a fmall Alteration of that Body's Diftance from the Earth; or the Force of Gravity, at all fmall Diftances from the Earth, acts equally on the fame Body. Suppofe then, the Time in which a heavy Body falls, to be divided into equal, but infinitely fmall Particles, and Gravitation acting in the firft Particle of Time, to give the Body an Impulfe towards the Center of the Earth, and make it ac-

quire a certain Degree of Velocity. Now, if, after that first Impulse, the Action of Gravity should cease, yet the Motion arising from the said Impulse would be continued, and the Body would (by 1 *Max.*) move uniformly, or with the same Velocity, towards the Centre of the Earth. But since Gravity acts in the second Particle of Time, with the same Force that it did in the first, the same Gravitation will give the Body another Impulse equal to the former; and so the whole Velocity, after these two Impulses, will be double of the first. If again the Action of Gravitation should cease, after the second Impulse, yet the Body would still move with two Degrees of Velocity. But since, in the third Particle of Time, the Body is yet urged by the same Force of Gravity as before, it will thereby acquire a third Degree of Velocity, equal to either of the other two. And in like Manner, in the fourth Particle of Time, it will acquire a fourth Degree; and so forth. Therefore, the heavy Body will (by 8 *Def.*) descend

with

with an uniformly accelerated Motion. W. W. D.

Corol. Hence, the Velocity acquired by the Fall of a Body from Rest, is always as the Time of the Fall.

Scholies.

1. IT may, in like Manner, from the same Principles be demonstrated, that, if a Body be forced directly upwards, it will move with an equably retarded Motion: Because the Force of Gravity still acting equally, contrary to the Body's Motion upwards, will in equal Times equally diminish that Motion, till it be totally destroy'd.

2. Though in the preceeding *Proposition* we have, to render the Demonstration the clearer, suppos'd the Body's Fall to begin from Rest; yet the Motion downwards will be an uniformly accelerated Motion, though we suppose its Fall to begin from any Degree of Velocity; by reason of the continual equal Impulses of Gravity.

Proposition XXI.

Theor. Fig. 22.

If one side AB *of a Triangle* ABC *represent the Time, in which a Body falls from Rest in a free Space, and another Side* BC *the Velocity acquired at the end of that Time; and through any Point* D *of* AB *there be drawn a right Line* DE *parallel to* BC: *Thus* DE *will represent the Velocity acquired at the end of the Time represented by* AD.

For (by reason of the similar Triangles ABC, ADE) as is AB : AD :: BC : DE. But BC represents the Velocity at the end of the Time AB: Therefore, since (by *Cor.* 20. *Prop.*) the Velocities are as the Times, DE will represent the Velocity at the end of the Time AD. W. W. D.

PROPO-

PROPOSITION XXIII.

Theor. Fig. 23.

THE Space run through in a certain Time, by a heavy Body falling from Rest, in a free Space, near the Surface of the Earth; is the half of that Space it would run through in the same Time, with the Velocity acquired in the last Instant of that Time.

Let AB be as the Time in which a heavy Body falls from Rest, and BC as the Velocity acquired at the end of it; complete the right-angled Triangle ABC. Suppose the Time AB divided into an infinite Number of equal Parts *Ar*, *re*, *ei*, *im*, *mp*, &c. and draw *ro*, *ef*, *ik*, *mn*, &c. parallel to the Base BC. Then (by 21. *Prop.*) *ro*, *ef*, *ik*, *mn*, &c. will represent the Velocities in the Particles of Time *re*, *ei*, *im*, *mp*, &c. And (by 1. *Prop.*) the Spaces run thro', in these Particles of Time, are as the said Velocities *ro*, *ef*, *ik*, &c. or (by 1. 6. *Eucl.*)

as the Parallelograms *eo*, *if*, *mk*, &c. Therefore the whole Space run thro', by the Fall of the heavy Body, in the Time AB, is as the Triangle ABC compos'd of all the infinitely little Parallelograms *eo*, *if*, *mk*, *pn*, &c. But if the Body was carried with the Celerity BC during the whole Time AB, the Space run thro' in that Time, would (by 3. *Cor.* 6. *Prop.*) be as the Rectangle AB X BC, which is double of the Triangle ABC. Therefore the Space run thro', by the Fall of a heavy Body from Rest, in the Time AB, is half the Space that it would have run thro' in that Time, with the Celerity acquir'd at the end of that Time. W. W. D.

8. *Corollaries.*

1. HENCE, as the *Space* descended from the Beginning of the Fall, in the Time AB, is represented by the Triangle ABC; so the Spaces descended, in the Times AP, AM, may be represented by the Triangles APQ, AMN. Hence,

2. The

2. The Spaces deſcended from the Beginning of the Fall, are in a duplicate Proportion of the Times of Deſcent. For the Spaces deſcended in the Times A B, A *p*, are as the Triangles A B C, A *p q*, which being ſimilar are in a duplicate Proportion of the Sides or Times A B, A *p*. Hence,

3. If the Times of a heavy Body's Fall from Reſt be as 1, 2, 3, 4, 5, *&c.* the Spaces deſcended in thoſe Times will be as 1, 4, 9, 16, 25, *&c.* the Squares of thoſe Numbers. Hence,

4. If the Times of Deſcent A *r*, A *e*, A *i*, A *m*, *&c.* be as 1, 2, 3, 4, *&c.* the Spaces deſcended in the Times A *r*, *r e*, *e i*, *i m*, *&c.* will be as the odd Numbers 1, 3, 5, 7, 9, *&c.*

5. Since the Velocities acquired by the Falls from Reſt, are (by *Cor.* 2. *Prop.*) as the Times; the Spaces deſcended from Reſt will (by 2 *Cor.* 22 *Prop.*) be in a duplicate Proportion of the Velocities acquired at the Ends of the Times.

A Scholy.

WE may, just as in 20. *Prop.* prove, that if a Body be constantly urged in any Direction, by a Force acting equally; its Motion will be an equably accelerated one: And consequently, as in 22 *Prop.* that the Space it runs through from the Beginning of the Motion, is half the Space it would run thro', in the same Time, with the Velocity acquired in the last Instant. From whence we may infer (after the same Manner as we inferred 2 *Cor.* from 22 *Prop.*) that the Spaces run thro', from the Beginning of the Motion, are in a duplicate Proportion of the Times.

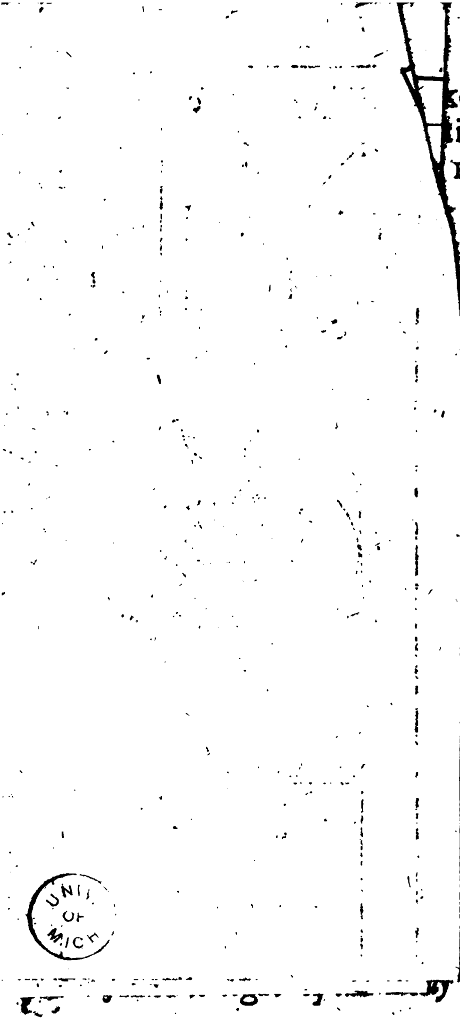
PROPOSITION XXIII.

Theor. Fig. 24.

IF a Body A *be held immoveable by three Powers, or Forces urging it according to the Directions* AB, AC, AE; *these Powers*

Powers will be to one another as three right Lines A D, A C, C D, *making a Triangle, whose first Side* A D *is a Part of the first Direction* A B *produc'd, the second Side* A C *the same with the second Direction* A C, *and the third Side* C D *parallel to the third Direction* A E.

For if the Body A be held immoveable by two Powers, or Forces, urging it in contrary Directions A B and A D, these two Forces will be equal; and so each of them may be represented by one and the same determinate Line A D. But (by *Hyp.*) the Body A is kept immoveable by three Forces urging it in the Directions A B, A C, A E: Therefore the joint Force of the latter two, must be equivalent to the first alone which is as A D, and must united urge the Body in the Direction A D with a Force as AD. But (by 3 *Cor.* 14 *Prop.*) two Forces which are as A C and A E, are united equivalent to the Force as A D urging in the Direction A D, and consequently to the Force as A D urging in the Direction A B. Therefore the three Forces



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Sine of the Angle of Inclination of that Plane to the Horizon is to the Radius.

Let CD represent the Horizon, and the Angle ACD will be the Inclination of the Plane; from A and B to CD demit the Perpendiculars AD and BE; through B draw a right Line HBF perpendicular to the Plane AC, and from F raise FG perpendicular to CD. The Body B is urged by three Powers, and kept immoveable by them: The First is the Body's absolute Force of Gravity, acting according to the Direction BE perpendicular to the Horizon CD; the Second is the Power, or Force R, urging the Body in the Direction BR parallel to the Plane; and the Resistence of the Plane, urging the Body according to the Direction BH, supplies the Place of the third Power. Therefore FG, QG, QF being respectively parallel to the Directions of these three Powers; there is (by 23 *Prop.* and its *Cor.*) as the Power R to the Body's absolute Gravity, so QG to FG :: (by 2 *Cor.* 8. 6. *Eucl.*) FG : GC :: (by 4. 6. *Eucl.*) AD :

AC ::

A C : : Sine of Inclination to the Radius. W. W. D.

A Corollary.

SINCE the Power R hinders the Descent of the Body B on the Plane A C, and is equivalent to the Body's Moment whereby it endeavours to descend; it is manifest, that a Body's Force to descend on an inclining Plane is to its absolute Force of Gravity, whereby it endeavours to descend in the Perpendicular to the Horizon, as the Sine of the Plane's Inclination is to the Radius.

Proposition XXV.

Theor. Fig. 26.

THE Descent of a heavy Body upon an inclining Plane, is an equably accelerated Motion.

Let B be a Body descending on the inclining Plane A D. Then (by *Cor. 24 Prop.*) the Force, whereby B endeavours

vours to deſcend on the Plane A D, is to its abſolute Force of Gravity, as the Sine of the Plane's Inclination A D C is to the Radius; which is a conſtant and invariable Proportion: And conſequently, the abſolute Force of Gravity being ſtill the ſame in the Body B, its Force, whereby it endeavours to deſcend upon the inclining Plane, remains alſo ſtill the ſame. Therefore this laſt Force will always act equally on the Body B; and conſequently its Deſcent on the inclining Plane A D, will eaſily be proved to be an uniformly accelerated Motion, by the ſame way of Reaſoning that was uſed in 20 *Prop.*

Corollaries.

1. HENCE, the Velocity acquired by the Deſcent of a heavy Body from Reſt on an inclining Plane, is always as the Time of Deſcent.

2. From 25 *Prop.* and 1 *Cor.* it is evident, that whatever has been demonſtrated in 22 *Prop.* and its *Corollaries*, of

heavy Bodies falling perpendicularly, is after the ſame Manner demonſtrable of their Deſcent upon inclining Planes. *Namely*, that the Space run thro' in a given Time by a heavy Body on an inclining Plane, computed from the Beginning of the Motion, is half the Space that it would uniformly run thro' in the ſaid Time with the Velocity laſt acquired. Alſo that the Spaces run thro' from the Beginning of the Motion, are in a duplicate Proportion of the Times, and alſo of the laſt acquired Celerities.

3. The Aſcent of a heavy Body upon an inclining Plane, is an uniformly retarded Motion.

PROPOSITION XXVI.

Theor. Fig. 26.

THE Velocity that a heavy Body B *acquires deſcending from Reſt on an inclining Plane* A D, *in any given Time, is to the Velocity that it would acquire in the ſame Time, falling from Reſt perpendicularly*

cularly in A C, *as the Plane's Height* A C *is to its Length* A D.

For the Increments of Velocity of the Body B, deſcending from A, in the Perpendicular A C, and on the inclining Plane A D; produced in an infinitely ſmall Particle of Time, are to one another as the Forces whereby they are produced: But theſe Forces are (by *Cor.* 24 *Prop.*) in the conſtant Proportion of the Radius to the Sine of the Plane's Inclination A D C; or as the Plane's Length A D to its Height A C: Therefore the Increments of the Velocities are in the Proportion of A D to A C. Therefore (by 12. 5. *Eucl.*) the Sum of the Increments in the Perpendicular A C is to the Sum of the Increments in the Plane, as A D is to A C; *that is*, the Velocity of the heavy Body, falling in the Perpendicular, is to the Velocity it would acquire, deſcending in the ſame Time on the inclining Plane, as the Length of the Plane is to its Height. W. W. D.

Proposition XXVII.

Theor. Fig. 27.

IF A B *represent an inclining Plane,* B C *an horizontal Line, and* A C *a Perpendicular hereto; then* C D *being drawn perpendicular to* A B, *I say, in the Time that a heavy Body descends, on the inclining Plane from* A *to* D, *that in the same Time the same or any other heavy Body would descend in the Perpendicular from* A *to* C.

If this be denied: Let A E be the Space run through on the inclining Plane, while A C is run through in the Perpendicular A C. Then because (by 22 *Prop.*) in the Time that the heavy Body descends from A to C, or from A to E, twice the Length of A C would be run through with an uniform Velocity, equal to that which is acquired in C by the perpendicular Descent, and so (by 2 *Cor.* 25 *Prop.*) would twice the Length of A E be run through with the Velocity acquir'd in E: The Velocity acquir'd in C will (by 1 *Prop.*) be to the Velocity acquir'd in E, as twice A C to twice A E,

or

or as AC to AE. But ſince AG and AE are Spaces deſcended in the ſame Time; therefore (by 26 *Prop.*) the Velocity in C is to the Velocity in E, as AB is to AC. Wherefore (by 11. 5. *Eucl.*) as is AB : AC :: AG : AE. But (by 2 *Cor.* 8. 6. *Eucl.*) as is AB : AC :: AC : AD. Therefore as AC : AE :: AC : AD; and conſequently AE is = AD. W. I. A.

Corollaries.

1. FROM this Demonſtration it is evident, that the Velocity acquired, in the ſame Time, by heavy Bodies deſcending from Reſt in the Perpendicular and on the inclining Plane, are as the Spaces run through by them.

2. Hence is found the Space through which a heavy Body falls in the Perpendicular, in the ſame Time that another Body deſcends a given Length AD on the inclined Plane AB; *namely*, if from the Point D there be raiſed to AB a Perpendicular DC, meeting the Perpendicular AC to the Horizon in C, AC will be the Space ſought.

PART II.

Of Centripetal Forces.

PROPOSITION XXVIII.

Lem. Fig. 28.

IF two right Lines AB *and* DB *meet in* B, *and touch a Circle in* A *and* D; *then* SB *being drawn from the Center* S, *also* DC ∥ SB, *and* AB *be produc'd till it meet with* DC *in* C: *I say, that* BD *is* = BC.

For SA and SD being drawn, SA is = SD; and (by 2 *Cor.* 36. 3. *Eucl.*) BA is = BD; also SB is a common Side of the Triangles ASB and BSD: Therefore (by 8. 1. *Eucl.*) is ∠ ABS = ∠ SBD: But (by 29. 1. *Eucl.*) ∠ SBD is = ∠ BDC, and ∠ ABS is = ∠ C: Therefore is ∠ BDC = ∠ C; and consequently (by 6. 1. *Eucl.*) BD = BC. W. W. D.

Corol.

Corol. Hence $\angle$ ABS $=$ SBD is $= \frac{1}{2}$ ABD.

PROPOSITION XXIX.

Lem. Fig. 29.

LET BEHIK *be a regular Polygon described about a Circle*, that is, *a Polygon of equal Angles, and equal Sides whereof every one touches the Circle: And let a Body in* A *be impelled in the tangential Direction* AC *by one single Impulse, and move on uniformly till it come to* B*; where, let it receive another single Impulse in the Direction* BS *from a centripetal Force tending to the Circle's Center* S*, such as being conjoined with the foresaid tangential or projectile Force in* B*, may turn the Body from the Tangent* AC *and make it move along the Tangent* BE. *When the Body is come to* E*, let it there receive a second Impulse from the centripetal Force directed to* S*: I say, it will then move along the third Tangent* EH. *When it comes*

to H, *let it receive a third Impulse from the centripetal Force : Then will it move along the fourth Tangent* H I: *And so forth. And the Body's Velocity through every Tangent will still be the same.*

For D C being drawn parallel to S B, B D is (by 28 *Prop.*) = B C, and (by 2 *Cor.* 14 *Prop.*) the Body's Velocity in B D (or B E) is equal to the Velocity it would have had in B C before the first centripetal Impulse in B, or the Velocity it had in A B. When the Body comes to E, it receives the second centripetal Impulse, which is equal to the former, because S E is = S B: For (by *Cor.* 28 *Prop.*) > S B D is = $\frac{1}{2}$ A B D = (by *Hyp.*) $\frac{1}{2}$ D E G = (by *Cor.* 28 *Prop.*) S E D; and (by 18: 3. *Eucl.*) > B D S is = a Right = S D E, and S D common: Therefore (by 26. 1. *Eucl.*) is S B = S E; in like Manner is S E = S H. Therefore, since the Body's Velocity along the Tangent B E, is the same with its Velocity along the Tangent A B, and the second centripe-

tal

tal Impulſe at E is equal to the firſt centripetal Impulſe at B; the ſecond centripetal Impulſe muſt draw it juſt as much aſide from the Tangent B E or D E, as the firſt did from the Tangent A B; *that is*, the Line of Direction, after the ſecond Impulſe, muſt make an Angle with B E equal to the Angle A B E; but (by *Hyp.*) > B E H is = > A B E: Therefore the Tangent E H muſt be the Body's Direction after the ſecond centripetal Impulſe; for this Direction muſt be between E S and E F: And G F being drawn parallel to E S, E G will (by 28 *Prop.*) be equal to E F; and (by 2 *Cor.* 14 *Prop.*) the Velocity in E G or E H will be the ſame with what it was in D E or B E and A B. In like Manner, after the third Impulſe of the centripetal Force in H (which is equal to either of the former in B or E, by reaſon that S H is = S E = S B) the next Direction will be in the Tangent H I with the ſame Velociry as before. And ſo on in the reſt of the Tangents. W. W. D.

A

A Scholy.

IN the preceeding Demonstration we only suppose, that the Center of the centripetal Force is the Center of a Circle, and that the Impulses of the said Force at equal Distances from the said Center are equal; which is a very possible and simple Hypothesis.

PROPOSITION XXX.

Theor.

A Body may move in the Periphery of a Circle by a projectile Force once imprest, and an uniform centripetal Force constantly imprest.

For a Body moved by a projectile Force once imprest, and a centripetal Force acting equally by an infinite Number of Impulses one after another, may (as is evident from 29 *Prop.*) run in the Perimeter of a regular Polygon described about a Circle. But the said Polygon's Peri-

Perimeter, when its Sides are infinite in Number, coincides with, and is nothing different from the Periphery of the Circle; and the said centripetal Impulses, when infinite in Number, are the same with an uniform centripetal Force acting constantly without Intermission. Wherefore it is evident, that a Body once impell'd by a projectile Force, and a centripetal Force acting constantly, may move in the Periphery of a Circle. W. W. D.

PROPOSITION XXXI.

Theor.

A Body moved in the Periphery of a Circle by a projectile Force once imprest, and an uniform centripetal Force constantly imprest, is constantly carried with an equable Motion; or its Velocity is still the same.

This is evident from 29 *Prop.* since the said centripetal Force is all one with a centripetal Force acting by an infinite Number of equal Impulses, and the Velocity

locity in the Perimeter of the circumſcribed regular Polygon of an infinite Number of Sides (which is the ſame with the Circle it ſelf) is by that *Prop.* ſtill the ſame.

A Corollary.

From 30 *Prop.* it is evident, that Mr. *Gordon*'s firſt *Theorem* (in his *Remarks* upon the *Newtonian Philoſophy* Pag. 27.) whereby he pretends to prove, that a Body impell'd by a projectile and a centripetal Force, conſtantly approaches to, or conſtantly recedes from the Center of the centripetal Force, is falſe and groundleſs: But this ſhall be farther proved below. It is alſo evident from 31 *Prop.* that all he ſays in *Pag.* 86 and 87 of his *Remarks*, about the conſtant Increaſe of Velocity of a Body moving in a Circle, is perfectly precarious and abſurd.

Scholy

Scholy. I.

IF our *Remarker's* foresaid first *Theorem* was true, it would indeed overturn the main Foundation of a great part of the *Newtonian Philosophy:* Therefore, that the *Remarker's* Mistake or Sophistry, and the Falshood of his Theorem may the better appear, we shall here set it down with his Demonstration; and plainly shew the Demonstration to be so faulty and inconsistent, that instead of proving the pretended Theorem, it proves nothing at all. His Theorem then is as follows in his own Words. See *Fig.* 30.

A Body turned from a streight lined Motion into a Curve that lies in a Plan, and describes, round any Point s, *Areas proportionable to the Times, by a continuing centripetal Force directed to* s; *must either move constantly away from* s, *or come constantly nearer to it, in a spiral Line.*

Let it here be observed, before we proceed further, that that Clause of this Theorem, which mentions the propor-

tionality of the Areas to the Times, is ſuperfluous, ſince he makes not the leaſt uſe of it in his Demonſtration. So that the ſaid Theorem ſhould be expreſſed thus: *A Body turned, by a continuing centripetal Force directed to* s, *from a ſtreight lined Motion into a Curve that lies in a Plan; muſt either move conſtantly away from* s, *or come conſtantly nearer to it, in a ſpiral Line.*

His Demonſtration, in his own Words alſo, is this.

Let the Body's ſtreight-lined Motion be the Line ABF, let the centripetal Force directed to s, be ſuppoſed to act upon the Body at the Point B; the Line BF in which the Body at B would run out, muſt make with BS, the Line of the Direction of the centripetal Force acting in B, either a Right, an Acute, or an obtuſe Angle. If the Angle SBF be a right or an acute Angle, then is the Angle SBC (a Part of that Angle) leſs than a right Angle. Draw out BC in infinitum towards Q and P, let us with our Authors, conceive the Point C to be the next

next Point in which the centripetal Force acts; which being supposed to act continually, or in every Point, the Point c must be the next Point to B in the Line B Q. Drop a Perpendicular from S, upon the Line P Q, it will fall somewhere betwixt B and Q; either upon C the next Point to B in that Line, or upon some other Point betwixt C and Q; suppose upon the Point N. If the Perpendicular fall upon C, then is C the nearest Point in the Line P Q to the Point S, nearer, to wit, than the Point B; if the Perpendicular fall upon N, then is C nearer to S than B, because nearer to N: So that the centripetal Force meeting with the Line in which the Body would run out at right Angles, or less than right Angles, must necessarily force the Body to move from a Point more distant to a Point less distant from the Center S. The Line C Q is that Line in which the Body at C would run out next, and the Point C is the next Point in which the centripetal Force is supposed to act; but a Perpendicular from S upon P Q fell, as above,

 either

either upon C, and then the Force directed to S, acting in the Point C, is at right Angles with CQ, or upon N, and then is this Force at leſs than right Angles with CQ. And ſo by the ſame reaſoning, if the Force directed to the Center be at any Time at right, or leſs than right Angles with the Line in which the Body would run out, it muſt ſtill continue to be ſo; and while it is ſo, the Body muſt move every Time the Force acts, or continually, from a Point more diſtant from S, to a Point leſs diſtant from S. But if a Curve be ſuch, that the Lines in which the Body would run out, *viz.* the Tangents of the Curve, make with the Radius conſtantly more than right Angles; then is the Body that moves in that Curve, moving conſtantly away from S: Or if the Tangent and Radius of this Curve at any Time be at right Angles, or leſs than right Angles, then muſt the moving Body come conſtantly nearer to the Center S in a ſpiral Line. Q. E. D.

This

This Demonstration is pretty cunningly contriv'd, but Fallacy all over: I shall discuss the first Part, wherein its main Strength lies, and the second, which is only hinted, will fall of Course. The said Demonstration then, supposes the centripetal Force to act upon the Body at the Point B, and then at the Point C next to B; now if the Points B and C be next to one another, either there is no Distance between them, or there is some Distance: If there be no Distance between B and C, then B and C coincide and really make but one Point, and so are equally distant from S, and consequently SC is = SB. But if there be some Distance (though ever so small) between the Points B and C, as the Demonstration plainly implies; for it supposes BC to be a right Line, SBC an acute Angle, and SCB a right one, from whence it infers the Point C to be nearer the Point S than the Point B is, which is indeed a very just Inference from the said Supposition, as is evident from 19. 1. *Eucl.* I say, if there be some Distance between the Points B and C, then C is

not the next Point to B in which the centripetal Force acts, since it acts continually, and consequently has acted in an Infinity of Points in the Time that the Body has moved from B to C, and the Body's Path has not been a right Line, as the said pretended Demonstration supposes, but a Portion of a Curve (as being produced by a projectile and constant centripetal Force) which we may represent by the Line BC, and allow it, being extremely small, to be very little different indeed from a right Line, yet still a Curve. Now since BC is really a Curve, though we suppose SC to be perpendicular thereto, we cannot therefore certainly infer that SC is shorter than SB, and consequently that C is nearer S than B is: For though SC be perpendicular to the Curve BC, *that is*, to the Tangent at the Point C, it may for all that be shorter or longer than, or equal to SB, according to the Nature of the Curve BC. For, for Instance, if BC be a Portion of a Circle whose Center is S, then SC perpendicular to BC is e-

qual

qual to S B; if B C be a Portion of an Ellipse whose Center is S, and S C half the shorter A X, which certainly is perpendicular to the Curve, then S C is shorter than S B; and if S C be half the longer A X, which also is Perpendicular to the elliptick Curve, then S C is longer than S B.

It is easy also to shew, that though S C be not perpendicular to the Curve B C, yet for all that, S C may be shorter or longer than S B. For if we suppose B C to be a very small Portion of an Ellipse, whose Center is S, and S B to be half the longer A X which therefore is perpendicular to B C, but S C is not perpendicular thereto, but oblique, because oblique to the Tangent in C; in this Case, S C is shorter than S B: Neither is it material here, whether we call the Angle S B C a right Angle or an acute one, it being a mixtilinear and not a rectilinear Angle. Again, if S B be half the shorter A X, and so also perpendicular to B C, S C is oblique to B C, and yet S C is longer than S B. Therefore that part of the preceeding Demonstration;

ſtration, where the Perpendicular is ſuppoſed to fall upon N, on purpoſe that SC may be oblique to B C, is of no Force or Moment imaginable. From all which (though we ſhould add no more) it is abundantly evident, that the ſaid Demonſtration is ſo far from proving *Mr. Gordon*'s Theorem, that it really proves nothing at all. Beſides, it is inconſiſtent with it ſelf, one Suppoſition deſtroying another.

But *laſtly*, perhaps it will be ſaid, that, ſince a Curve may be conſidered as compos'd of an infinite Number of right Lines, the Line B C, the Body's Path from B to C, is an infinitely little right Line: This indeed we can eaſily allow, ſuppoſing the centripetal Force not to act at all between the Points B and C, but not otherwiſe; for if it act between B and C, it muſt either act conſtantly or by Starts; if conſtantly, then B C muſt be a perfect Curve, as above, though infinitely little different from a right Line, and ſo the pretended Demonſtration is already overthrown; if by Starts, then B C muſt be an inflected Line, and not

a ſtreight one, though yet infinitely little different from a ſtreight one. Wherefore it is plain, that, if B C be a ſtreight Line, as the Demonſtration plainly ſuppoſes, the centripetal Force acting upon the Body in the Point B, does not act upon it again till it comes to the Point C. Now it is aſſerted, that a Perpendicular from S to P Q, or B C produc'd, muſt either fall upon C, or upon ſome other Point between C and Q: But I wonder how Mr. *Gordon* is ſure of this, ſince, I believe, any Body but himſelf will grant, that the Perpendicular from S to P Q may poſſibly fall upon ſome Point between B and C, as well as upon C, or between C and Q: And if it fall between B and C, as certainly it may, becauſe, be the Line B C ever ſo ſhort, the Angle C B F may be ſo extremely little, or B C may ly ſo extremely cloſe to B F, that the Angle S C B may be acute, as well as the Angle S B C; then S C may either be equal to, or longer or ſhorter than S B; which again ſpoils the fine Demonſtration.

Scholy II.

AN Infinity of Kinds of centripetal Forces may be conceived, or are possible, each constantly acting in a certain regular Manner, or by a constant Rule or Law: Though the Law whereby one acts, be different from the Law whereby another acts.

As for Instance, there may be one kind of centripetal Force that may act upon a Body in such Manner, that its Impulses may constantly be directly as the Distances of the Body from the Center of the centripetal Force: Another Kind of centripetal Force may so act, that its Impulses may constantly be, as the Distances reciprocally: A third may so act, that its Impulses may constantly be as the Squares of the Distances reciprocally; and so forth. Of the third Kind it is highly probable, that there are a great many in Nature.

If v be put for the Energy, Efficacy, or Impulse of the centripetal Force, D for

for the impelled or attracted Body's Distance from the Center of the centripetal Force; then the Law of the first mentioned Kind of centripetal Force is, that V is every where as D directly: The Law of the second Kind is, that V is still as D reciprocally: And the Law of the third is, that V is every where as D^2 reciprocally. There may be a fourth Kind of centripetal Force conceiv'd, that acts equally or alike at all Distances: And so of others to Infinity.

Now 'tis plain, that an Infinity of Kinds of centripetal Forces is possible, each acting by one fixt and constant Law, the Conception of this involving no Impossibility or Contradiction.

Proposition XXXII.

Theor. Fig. 31.

IF *a Body once impell'd by a projectile Force in any right Line* a f, *be turned from it, by a constant centripetal Force directed to any fixt Point* s, *not lying*

lying in the Line a f; *the Body will move in a Curve lying in the Plane that passes through* s, *and the Length of* a f; *the Curve will also be concave or hollow towards* s, *and the Body will describe round* s *Areas proportional to the Times of describing.*

Suppose the Body, in any Particle of Time, to describe by the projectile Force, the Line *ab*; it would, if nothing hindred it, in an equal Time, describe *bf* = *ab*: And if it be urged in *b*, by the centripetal Force, in the Direction *b*s, it will (by 14 *Prop.*) in an equal Time, describe the Diagonal *bc* of the Parallelogram *bgcf*. The △*a*s*b* is = △*bsf*, by 38. 1. *Eucl.* because *ab* is = *bf*; also △*bsc* is = △*bsf*, by 37. 1. *Eucl.* because *fc* is ∥ *bs*: Therefore is △*bsc* = △*asb*. Now these Triangles *bsc*, *asb* are Areas described in equal Times; and by the same Way of Reasoning, the centripetal Force directed to s acting again at the Point *c*, must oblige the Body to describe the Area *c* s *d* equal to the Area *b sc*, in a third Time equal

to either of thoſe in which it deſcribed the Area *asb* or *bsc*; and ſo of the Area *dse*, or any Number of ſuch Areas: And if any two Numbers of ſuch Areas be taken, their Sums will be to one another as the Sums of their Times; or if theſe Areas be proportionally diminiſhed, they will ſtill be to one another as before. Let us therefore now ſuppoſe the centripetal Force not to act by Starts, but conſtantly without Intermiſſion; and conſequently that the Lines *ab*, *bc*, *cd*, &c. and the Areas comprehended within theſe Lines are diminiſh'd to Infinity, ſo as to bring the Points *b*, *c*, *d*, *e*, &c. in which the centripetal Force is ſuppoſed to act, next to one another; then will the Line *abcde* turn into a Curve, and the Areas that the Body will deſcribe, round s in this Curve, will be proportional to the Times of Deſcription, as above. And 'tis evident alſo, that the Curve will ly in a Plane paſſing through s and *ab*, and will be concave towards s, w. w. d.

A Scholy.

THE preceeding *Prop.* is Sir *Isaac Newton's* 1 *Prop.* 1 *Lib. Princip.* and Dr. *Gregory's* 11 *Prop.* 1 *Lib. Astron.* and I have expressed and demonstrated it very near in Mr. *Gordon's* Words, the Demonstration, as he gives it, being just enough. But here Mr. *Gordon* very unreasonably finds Fault with those two great Men (see *Pag.* 25, 26, 27 of his *Remarks*) because they have not determined the Species of the Curve, that the projectile and centripetal Forces oblige the Body to describe, though he shews not (as I'm confident he cannot) the least Flaw in the Demonstration. It is true indeed, that neither *Newton* nor *Gregory* determines the Kind and Nature of the Curve, for in this Place they could not, the Thing being absolutely impossible; since they suppose only any Kind in general of a centripetal Force acting, and not any particular Species of centripetal Force acting by one constant and

par-

ticular Law: So that the Curve may be any one amongſt an Infinity of Curves. And the Nature of the Curve muſt depend upon the Nature of the centripetal Force, and the Quantity of the projectile Force, whereby it is deſcribed. *Gordon* may juſt as reaſonably quarrel with a Perſon treating only of the Properties of a Parallelogram in general, without deſigning to deſcend to any particular Sort of Parallelogram; that he did not determine the particular Sort of the general Parallelogram, with which alone he was at that Time concerned. Now how unreaſonable and abſurd this would be, we leave every one to judge.

PROPOSITION XXXIII.

Theor. Fig. 31.

EVERY Body moving in a Curve, and deſcribing round another Body Areas proportional to the Times of Deſcription, is forced into ſuch a Curve by a projectile Force once impreſt, and a centripetal Force

constantly directed to that Body, round which the Areas are describ'd.

Any Curve may be conceived as made up of an Infinity of right Lines; suppose the revolving Body to move in the inflected Line *abcde*, and let this Line be compos'd of such infinitely small right Lines *ab*, *bc*, *cd*, &c. as the Body describes in equal Particles of Time. Let the single projectile Impulse be made at *a*, in the Direction *ab*; and let s be the Point or Body at Rest, about which the other Body revolves. Produce *ab* to *f*, till *bf* be = *ab*, and draw *as*, *bs*, *fs*, *cs*, &c. The Body in *a*, in any certain Time, describing by the projectile Force the Line *ab*, would, if nothing hindred it, describe, in an equal Time, *bf* = *ab*; and since it describes not *bf*, but *bc*, it is turned from moving in *bf* by some Force acting in *b*, different from the projectile Force. The △ *asb* is (by 38. 1. *Eucl.*) = △ *bsf*; the △ *bsc* is (by *Hyp.*) = △ *asb*, because these are Areas described in equal Times: Therefore is △ *bsc* = △ *bsf*: Whence (by

39.

39. 1. *Eucl.*) fc is ∥ bs. Therefore the Direction of the Force that turn'd the Body from moving in the Line bf, to move in the Line bc, is (by 15 *Prop.*) parallel to cf, and consequently is the Line bs. By the same way of Reasoning it is plain, that the Force (that turns the Body from a straight Course) acting in all the other Points c, d, e, &c. may also be proved to be directed to s.

The same Demonstration holds good, when both the Body s and the revolving Body, are urged by equal accelerating Forces according to parallel right Lines, *that is*, when both are urged with equal Velocity in a straight Course; as will be evident from the first *Scholy* following.

Scholy I.

IF Bodies be moved anywise among themselves, and afterwards be urged by equal accelerating Forces, according to parallel right Lines, or (which is all one) if their common Center of Gravity move in a right Line: They will all continue

to move after the same Manner as at first, in respect of one another, as if they were not urged by the said Forces.

For the said equal accelerating Forces, acting in parallel Directions, will move all the Bodies with equal Velocities in the said Directions; and will therefore never change the Positions and Motions of the Bodies in respect of one another; and consequently they will continue to move in respect of one another, after the same Manner as before.

Scholy II. *Fig.* 31.

THE last *Prop.* is Sir *Isaac Newton's* 2 *Prop.* 1 *Lib.* and Dr. *Gregory's* 12 *Prop.* 1 *Lib.* All that Mr. *Gordon* says to it, in *Pag.* 14, 15, 16, 17, 18, 19, 20, 21, 22 of his *Remarks*, that is any-wise to Purpose, is, that *bogr* being a Parallelogram, the Force in *bg* directed to s may be resolved into the Forces *bo* and *br*, neither of which is directed to s, which indeed is very true; and so may the said Force be resolved into a thou-

sand

sand other Forces; and in that Case, the Sense of the Proposition is, that the Force, that is compounded of all these Forces, is directed to the Point s, as Sir *Isaac Newton* himself acknowledges and affirms in *Sch.* of his forementioned *Prop.* in these Words. *Urgeri potest corpus a vi centripeta composita ex pluribus viribus: in hoc casu sensus Propositionis est, quod vis illa quæ ex omnibus componitur, tendit ad punctum* s.*

But now, when we apply the preceeding Proposition to the real Phænomena of Nature, and not to artificial Things, as a Ship carried by a Stream, or animate Things, as a Man walking, which are Instances the *Remarker* very impertinently brings in 15 *Pag.* is it not far more reasonable (since, according to 10 *Max.* Nature acts by the simplest Methods) to suppose, that the Force directed to s is the single one in *bg*, than two in *bo*, and *br* compounding the Force in *bg*. The last Case of the two Forces is indeed very possible, but the first of the single Force is far more probable: Since 'tis

'tis highly improbable that Nature uſes a compound way, where a ſimple way will do as well. If then the Planets revolve round the Sun in Curves, and deſcribe Areas proportional to the Times, as is well known by a Multitude of aſtronomical Obſervations; muſt we not conclude, that each of them is urged by a ſimple centripetal Force (and not a compound one) directed to the Sun, though not with abſolute mathematical Certainty? If we ſuppoſe the direct centripetal Force to be compounded of other Forces, in different Directions running quite away from the Sun, in one Place of a planetary Orbit, we have the ſame Reaſon to ſuppoſe the like in all other Places thereof. Now what a needleſs, fooliſh, and unreaſonable Compoſition of Forces does this ſeem to be? And what imaginable Ground can there be for ſuch an abſurd Fancy, though this be not ſimply and abſolutely impoſſible?

PROPO-

Proposition XXXIV.

Theor. Fig. 32.

A Body impell'd by a projectile Force once imprest, and a centripetal Force constantly imprest, may move in any Curve that is concave to the Center of the centripetal Force.

This, I think, is pretty near the Reverse of, and almost quite contrary to our *Remarker's* first *Theorem*, which pretends the Curve to be a spiral Line only.

Let any Curve, that is concave to the Center of the centripetal Force, be circumscribed by many Tangents A B, B C, C D, D E, &c. produc'd to H, I, K, &c. and let S be the Center of the centripetal Force: Suppose a Body in A be impelled by one single projectile Impulse, in the Direction of the Tangent A B; and after it has run from the Contact A to B, let it receive, in B, a first single Impulse from the centripetal Force directed

to

to S, ſuch as may turn it from the firſt Tangent A B or A H into the Direction of the ſecond Tangent B C: For (by *Sch.* 15 *Prop.*) we may conceive a Force tending to S, in the Direction B S, ſo adjuſted and proportioned to the Force in A H or B H, as may make the Body move in any middle Direction B C between B H and B S. When the Body has run in the ſecond Tangent B C beyond the Contact B to the Point C, let it get a ſecond Impulſe from the centripetal Force tending to S, and that ſuch as may (by *Sch.* 15 *Prop.*) turn it into the Direction of the third Tangent C D between C I and C S: When it has run beyond the Contact G to ſome other Point D, let a third centripetal Impulſe turn it into the Direction of a fourth Tangent D E; and ſo on. Now ſuppoſe the Number of centripetal Impulſes, and the Number of Tangents to be each multiplied to Infinity; then the infinite Number of Impulſes of the centripetal Force, will be equivalent to a conſtantly continuing Impulſe, or a centripetal Force acting inceſſantly; and the

infinite

infinite Number of Tangents (which will all be infinitely ſmall, becauſe the centripetal Impulſes are infinite in Number) will degenerate into, and be all one with the Curve it ſelf. From whence it is maniſeſt, that a Body impell'd by a projectile Force once impreſt, and a centripetal Force conſtantly impreſt, may move in any Curve which is concave to the Center of the centripetal Force. W. W. D.

A Scholy.

WE have here ſuppoſed any centripetal Force in general acting at Pleaſure, without determining any particular Law by which it acts; becauſe we have determined no particular Species of Curves. This we were obliged to do, that the Demonſtration might be the more general; and it is moſt evident, that the ſaid Demonſtration agrees to the Ellipſe, any Point within it being the Center of the centripetal Force; ſince the Ellipſe will then be one Species of ſuch Cuves.

But

But if the Center of the centripetal Force be the Center of the Ellipſe, or one of the umbilick Points, it is demonſtrated by Sir *Iſ. Newton*, that the centripetal Force will be a regular one, or ſuch as acts by one conſtant Law: In the former Caſe, his Law of the centripetal Force is, that its Impulſes are directly as the Diſtances of the revolving Body from the Center; ſee 10 *Prop.* 1 *Lib. Princip.* In the latter Caſe, when one Focus of the Ellipſe is the Center of the centripetal Force, his Law is, that the centripetal Impulſes are as the Squares of the Diſtances reciprocally, or (as that great Man expreſſes it) the centripetal Force is reciprocally as the Square of the Diſtance; ſee 11 *Prop.* 1 *Lib. Princip.* Sir *Iſ. Newton* alſo determines the particular Laws of the centripetal Force in ſeveral other Curves; all which he deduces from his general Law delivered and demonſtrated in his 6 *Prop.* 1 *Lib.* The Proof of this general Law, on which the reſt depend, alſo the Proof of the two particular Laws in the two forementioned

tioned Cafes, we fhall here largely deliver in the five following Propofitions; *viz.* The general Law, and that of the latter Cafe, after Dr. *Gregory*'s full and clear Method; but that of the former Cafe (which *Gregory* has not) after *Newton*'s own Method enlarged. *Newton*'s own Demonftration of his general Law we fhall refer to the end of this Tract.

PROPOSITION XXXV.

Lem. *Fig.* 33, 34, 35.

THE *nafcent, or evanefcent Subtenfe of the Angle of Contact in a Circle, is in a duplicate Proportion of the contermi- nal Arch.*

This is Doctor *Gregory*'s 24 *Prop. Aftron.*

Let ADC be a Circle, AB a Tangent in A, and fo the Angle BAD, made by by the Tangent AB and Arch AD, the Angle of Contact. I fay, that any Subtenfe thereof, infinitely near the Point A of Contact, as BD, is as the Square of

 the

the Arch A D; *that is*, this Subtenſe B D is to another Subtenſe *b d* infinitely near A, as the Square of the Arch A D is to the Square of the Arch A *d*; provided B D and *b d* be parallel as in *Fig.* 33 and 34, or, being produced, meet in ſome Point O remote from A, as in *Fig.* 35, ſo that they be infinitely near parallel.

Draw the Diameter A C, which (by 18. 3. *Eucl.*) will be perpendicular to the Tangent A B.

1 *Caſe.* Let the Subtenſes D B, *d b* be (in *Fig.* 33.) perpendicular to A B.

Draw D E, *d e* parallel to A B; draw alſo C D, C *d*, and the Chords A D, A *d*. The Arch A D being infinitely little or naſcent, does infinitely near coincide with its Chord A D; and conſequently is infinitely little different from, and ſo is to be conſidered as the ſame with the ſaid Chord or right Line A D: In like Manner does the Arch A *d* coincide with its Chord A *d*. But (by 31. 3. *Eucl.*) A D C is a right Angle; therefore (by 2 *Cor.* 8. 6. *Eucl.*) as is A C : Chord A D :: Chord A D : A E or B D. Therefore is Chord

A

$AD_q = BD \times AC$; and consequently $\frac{\text{Chord } AD_q}{AC} = BD$. Just so is Chord $Ad_q = bd \times AC$, and $\frac{\text{Chord } Ad_q}{AC} = bd$. Therefore is $\frac{\text{Chord } AD_q}{AC} : \frac{\text{Chord } Ad_q}{AC} :: BD : bd$; and consequently Chord AD_q : Chord $Ad_q :: BD : bd$. Therefore, since the Arch AD coincides with, and so is infinitely near equal to, the Chord AD, and the Arch Ad coincides with, and so is infinitely near equal to the Chord Ad; there will be as $BD : bd ::$ Arch AD_q : Arch Ad_q. W. W. D. 1.

2 *Case.* Let the Subtenses BD, bd (*Fig.* 34.) of the Angle of Contact be yet parallel, but not perpendicular to the Tangent AB.

Draw DF, df perpendicular to AB; then (by reason of the equiangular Triangles DFB, dfb) there will be as $BD : bd :: DF : df$. But (by 1 *Case*) as $DF : df :: AD_q : Ad_q$. Therefore as $BD : bd :: ADq : Adq$. W. W. D. 2.

3 *Case.* Let the Subtenses BD, bd (*Fig.* 35.) converge towards some remote

mote Point o, but so that they may be infinitely near parallel.

Draw DF, df perpendicular to the Tangent AB. Since BD, bd are supposed to be parallel, the ∠ FBD is = ∠ fbd, and consequently the Triangles FBD, fbd are equiangular. Therefore as BD : bd :: DF : df. But (by 1 *Case*) as DF : df :: ADq : Ad_q. Therefore as BD : bd :: ADq : Ad_q. W. W. D. 3.

A Corollary.

THE nascent or evanescent Subtense of the Angle of Contact, is also in a duplicate Proportion of the conterminal Arch, in any other Curve to which there may be described an equicurve Circle; or a Circle whose Curvature is the same with (or infinitely little different from) the Curvature of a small Portion of that Curve; such as are the Conic Sections, and many other Curves. For if the Circle AD (*Fig.* 34.) be of the same Curvature with a small Portion AD of the Curve ADG, the Points D and

d

d will both be in the Periphery of that Circle, and alſo in this Curve : Therefore the naſcent or evaneſcent Subtenſes BD, *bd* of the common Angle of Contact, will be in a duplicate Proportion of the conterminal Arches AD, A*d* of the Curve ADG, as well as of the Circle ADC.

PROPOSITION XXXVI.

Theor. Fig. 36.

If a Body be projected according to the Direction of any right Line PR, *and at the ſame Time be urged by a centripetal Force conſtantly tending to a Center* S, *ſo that by the compound Motion it deſcribe the Curve* APp; *if alſo, the right Line* PR *touching the Curve in any Point* P, *from another Point* B *in the Curve infinitely near* P *there be drawn the right Line* BD *perpendicular to the right Line* SP, *and* BR *parallel to* SP : *And the like Conſtruction be made at any other Point* p *of the Curve,* pr *being a Tangent in* p,

b infinitely near p, r b parallell and b d perpendicular to s p. Then the centripetal Force in P *will be to the centripetal Force in p as* $\frac{Spq \times bdq}{br}$ *is to* $\frac{SPq \times BDq}{BR}$; *or the centripetal Force in any Point* P *will be reciprocally as the solid* $\frac{SPq \times BDq}{BR}$, *when the Figure* PRBD *is infinitely little.*

This is the great Newton's *general Law of centripetal Forces, which the deservedly famous Dr.* Gregory *demonstrates after the following Manner.*

Let the Force or Impulse in P tending to the Point S, be called V; and let the Time wherein the infinitely little Arch PB is run thro' by the Body with the compound Force, or whereby the Body, by the projectile or natural Force alone, would run thro' the infinitely little Tangent PR, be called T.

Let the centripetal Force in *p* be called *v*; and the Time, in which the infinitely little Arch *p b* is run thro', or in which the Tangent *p r* would be run

thro', be called t. Let P c be an Arch run thro' in a Time equal to the Time t, in which the Arch pb is run through; and draw cf parallell to SP.

Then (by *Lem.* 5 *Prop.*) is $\frac{BR}{br} =$ $\frac{BR}{cf} \times \frac{cf}{br}$: But (by 35 *Prop.* and its *Cor.*) $\frac{BR}{cf}$ is $= \frac{PBq}{Pcq}$; and these Arches PB, Pc being infinitely little are (by 1. 6. *Eucl.*) as the Triangles BSP, cSP; *that is* (by 32 *Prop.*) as the Times in which they are described, or (by *Constr.*) as the Times in which the Arches PB, pb are described, or as T, t: Consequently $\frac{BR}{cf} = \frac{PBq}{Pcq}$ is $= \frac{T^2}{t^2}$. Again (by 2 *Max.*) the little Line cf is to the little Line br as the Causes that produce them, *that is*, as the centripetal Force in P to the centripetal Force in p, or as V to v: Consequently $\frac{cf}{br}$ is $= \frac{V}{v}$. Therefore is $\frac{BR}{br} =$ $\frac{T^2}{t^2} \times \frac{V}{v}$. Whence $\frac{V}{v}$ is $= \frac{BR}{br} \div \frac{T^2}{t^2} =$ $\frac{BR \times t^2}{br \times T^2} = \frac{BR}{br} \times \frac{t^2}{T^2}$. But (by 32 *Prop.*) as T to t so is Area SBP to Area sbp, or twice

twice SBP to twice sbp, or (by 41. 1. *Eucl.*) as $SP \times BD$ to $sp \times bd$. Therefore is $\frac{t^2}{T^2} = \frac{Spq \times bdq}{SPq \times BDq}$; and consequently $\frac{V}{v} = \frac{BR}{br} \times \frac{t^2}{T^2}$ is $= \frac{BR}{br} \times \frac{Spq \times bdq}{SPq \times BDq}$; whence V is to v (as $BR \times spq \times bdq$ is to $br \times SPq \times BDq$, or as $\frac{BR \times Spq \times bdq}{BR \times br}$ is to $\frac{br \times SPq \times BDq}{BR \times br}$ or) as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$. Therefore the centripetal Force in P (tending to S) is to the centripetal Force in p, as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$: Or (to express the Thing shorter) the centripetal Force in P, is reciprocally proportional to the nascent or evanescent solid $\frac{SPq \times BDq}{BR}$

W. W. D.

A Corollary.

HENCE, if any Curve APp be given, and a Point S to which the centripetal Force tends; the Value of the Solid $\frac{SPq \times BDq}{BR}$ may be determined from the Nature of the Curve; and consequently the Law of the centripetal Force, which is reciprocally as the said Solid, may be found.

PROP.

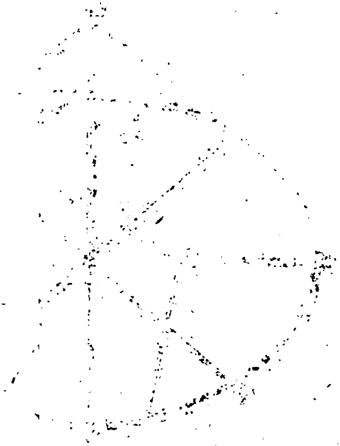
Proposition XXXVII.

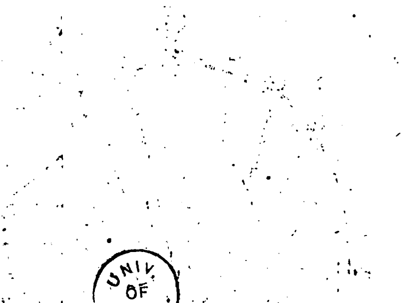
Lem. *Fig.* 37.

If a right Line R P Z *touch, in any Point* P, *an Ellipse* A P I, *whose umbilick Points or Foci are* S *and* F ; *and through the Ellipse's Center* C *there be drawn a Diameter* I K *parallel to* R Z: *Then, a right Line being drawn from* S *to* P, *that Part thereof* E P, *which is intercepted by the Parallels* I K *and* R Z, *is equal to* C A *half the longer Ax.*

Draw F P, and F H ∥ R Z or I K. Since (by 3. 4. of *Milnes's Conic Sections*) the Angles F P Z, H P R are equal ; also the Angles P F H, P H F alternate to these, are equal ; whence P H is = P F. Again, since S and F are the Foci, and C the Center, S C is = C F: Therefore (by 2. 6. *Eucl.*) S E is = E H. But S H is the Difference of P S and P H : Therefore S H is also the Difference of P S and P F, and E H the Half-difference of the same. Wherefore E P (made up of the lesser Quantity P F or

P H,





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Let

Let APM be the Ellipse in which the Body revolves, and let S be that Focus of the Ellipse to which the centripetal Force is directed. Draw the conjugate Axes TA, GY, crossing one another in the Center C. And at any Point P of the elliptick Curve draw a Tangent HPZ, draw also the Diameter PM, and ICK a conjugate thereto or parallel to the Tangent HZ, and to IK demit the Perpendicular PN. Complete the Parallelogram PCIH, and IH will touch the Ellipse in I. Joyn SP cutting IK in E. From the Point B infinitely near P draw BR parallel to SP, and Bx to RZ, meeting SP in x, and MP in o. Draw also BD perpendicular to SP.

The centripetal Force tending to S is (by 36 *Prop.*) reciprocally as the Solid $\frac{SPq \times BDq}{BR}$. This then must be computed from the Nature of the Ellipse. To which Purpose let L denote the Parameter of the longer Ax TA: Then (by *Lem.* 5 *Prop.*) is $\frac{L \times BR}{BDq} = \frac{L \times BR}{L \times Po} \times$

$\frac{L \times Po}{Mo \times Po} \times \frac{Mo \times Po}{Boq} \times \frac{Boq}{Bxq} \times \frac{Bxq}{BDq}$. But (by 1. 6. *Eucl.*) $\frac{L \times BR}{L \times Po}$ is $= \frac{BR}{Po} = \frac{Px}{Po} =$ (by 2. 6. *Eucl.*) $\frac{PE}{PC}$, because xo is $\parallel$ EC; and (by 37 *Prop.*) PE is $=$ AC. Therefore is $\frac{L \times BR}{L \times Po} = \frac{AC}{PC}$. Again, $\frac{L \times Po}{Mo \times Po}$ is $= \frac{L}{Mo}$. And (by 2 *Cor.* 20. 1. *Miln. Conic.* and *Permut.*) $\frac{Mo \times Po}{Boq}$ is $= \frac{MC \times CP}{ICq} = \frac{CPq}{ICq}$. And, in the present Case, Bo is $=$ Bx, the Point B being infinitely near the Point P: Therefore is $\frac{Boq}{Bxq} = 1$, and so $\frac{Boq}{Bxq}$ goes out of the first Equation. Farther $\frac{Bxq}{BDq}$ is $= \frac{PEq}{PNq}$; because the Triangles BDx, PNE are equiangular, for the Angles at D and N are Right, and the alternate Angles BxD, PEN of the two Parallels Bx, EN are equal. And PEq is $=$ CAq, because, as before, PE is $=$ CA: Therefore is $\frac{Bxq}{BDq} = \frac{CAq}{PNq}$. But (by 11. 2. *Miln. Conic.*) the Rectangle

angle GC $\times$ CA is equal to the Parallelogram PCIH or (by 35. 1. *Eucl.*) the Rectangle IC $\times$ PN; therefore (by 16. 6. *Eucl.*) as CA : PN :: IC : GC; whence as $CAq : PNq :: ICq : GCq$, therefore is $\frac{Bxq}{BDq} = \frac{ICq}{GCq}$. Therefore since, as before, the Proportion of L $\times$ BR to BDq is compounded of the Proportions L $\times$ BR to L $\times$ Po, L $\times$ Po to Mo $\times$ Po, Mo $\times$ Po to Boq, (Boq to Bxq which goes off) and Bxq to BDq; it must also be compounded of the Proportions respectively equal to these; *that is*, $\frac{L \times BR}{BDq}$ is $= \frac{AC}{PC} \times \frac{L}{Mo} \times \frac{CPq}{ICq} \times \frac{ICq}{GCq} = \frac{AC \times L \times CPq \times ICq}{PC \times Mo \times GCq \times ICq} = \frac{AC \times L \times CP}{Mo \times GCq}$. But (by 4 *Cor.* 24. 1. *Milns Conic.*) $AC \times L$ is $= 2GCq$. Therefore is $\frac{L \times BR}{BDq} = \frac{2GCq \times CP}{Mo \times GCq} = \frac{2CP}{Mo}$. But since the Points B and P are infinitely near one another, Mo is ($= MP$) $= 2CP$. Therefore is $BDq = L \times BR$; and consequently $\frac{SPq \times BDq}{BR} = \left(\frac{SPq \times L \times BR}{BR}\right) = SPq \times L$; Wherefore the centripetal

L Force

Force in P is reciprocally proportional to $SP_q \times L$, as is evident from 36 *Prop.* or, becaufe L is a conftant and invariable Quantity, the centripetal Force is reciprocally as SP_q. Therefore the centripetal Force tending to one of the Foci of an Ellipfe, is reciprocally as the Square of the revolving Body's Diftance from that Focus. W. W. D.

A Scholy.

At the End of this Demonftration it is proved, that the centripetal Force is reciprocally as $SP_q \times L$; that's to fay, the centripetal Force in the Point P is to the centripetal Force in any other Point p (when the Force is directed to a Focus S of the Ellipfe) as $Sp_q \times L$ is to $SP_q \times L$, or as Sp_q is to SP_q; which is all one as to fay, that the centripetal Impulfes or Forces in the Points P and p are reciprocally as the Squares of the revolving Body's Diftances from the faid Focus S.

PROPOSITION XXXIX.

Theor. Fig. 39.

IF a Body once impell'd by a projectile Force, and constantly urg'd by a centripetal Force, revolve in an Ellipse, the centripetal Force tending to the Center of the Ellipse: The Law of the said centripetal Force will be such, that its Impulses on the revolving Body, will always be directly as the Distances of the said Body from the said Center.

Let SA, SG be the Semi-axes of the Ellipse, whose Center S is also the Center of the centripetal Force; MP, IK two conjugate Diameters, PN, BD Perpendiculars to the said Diameters; B*o* an Ordinate to the Diameter MP, and consequently parallel to PR the Tangent at the Vertex P and to IK; complete the Parallelogram B*o*PR, and suppose the Point B to be infinitely near the Point P.

Then (by 2 *Cor.* 20. 1. *Miln. Conic.*) as is P*o* X *o*M: B*o*$_q$ :: SP$_q$: SI$_q$; and

L 2 (by

by reason of the similar Triangles D B O; P N) as $BO_q : BD_q :: SP_q : PN_q$. Therefore is $\frac{Po \times oM}{Bo_q} = \frac{SP_q}{SI_q}$, and $\frac{Bo_q}{BD_q} = \frac{SP_q}{PN_q}$. Whence $\frac{Po \times oM}{Bo_q} \times \frac{Bo_q}{BD_q} =$ (by *Lem.* 5 *Prop.*) $\frac{Po \times oM}{BD_q}$ is $= \frac{SP_q}{SI_q} \times \frac{SP_q}{PN_q} = \frac{SP_q \times SP_q}{SI_q \times PN_q}$. Therefore as is $Po \times oM : BD_q :: SP_q \times SP_q : SI_q \times PN_q$; and consequently $BD_q \times SP_q \times SP_q = Po \times oM \times SI_q \times PN_q$. Wherefore $\frac{BD_q \times SP_q \times SP_q}{Po}$ $\left(= \frac{BD_q}{Po} \times SP_q \times SP_q\right)$ is $= oM \times SI_q \times PN_q$: And consequently as $oM : \frac{BD_q}{Po} :: (SP_q \times SP_q : SI_q \times PN_q ::)\ SP_q :: \frac{SI_q \times PN_q}{SP_q}$. Complete the Parallelogram S P H I, and for P O put its Equal B R, and for $SI \times PN$ (or P H I) put $SG \times SA$ equal thereto by 1. 2. *Miln. Conic.* also the Points B and coming close together, for O M put S P: Then, multiplying the Extremes and Middles of the last Analogy, you will get $\frac{BD_q \times SP_q}{BR} = \left(\frac{2\,SP \times SG_q \times SA_q}{SP_q} =\right)$

2

$\frac{2SGq \times SAq}{SP}$. But (by 36. *Prop.*) the centripetal Force in P is reciprocally as the nascent Solid $\frac{SPq \times BDq}{BR}$: Therefore it is also reciprocally as $\frac{2SGq \times SAq}{SP}$. Now $2SGq \times SAq$ is a given or constant Quantity; therefore is $\frac{2SGq \times SAq}{SP}$ as $\frac{1}{SP}$. Whence the centripetal Force is reciprocally as $\frac{1}{SP}$, and consequently directly as the Distance SP. W. W. D.

A Scholy.

IN the former Demonstration it is proved, that $\frac{SPq \times BDq}{BR}$ is $= \frac{2SGq \times SAq}{SP}$. Now, if pr be a Tangent in p, rb parallel to sp, and bd perpendicular to sp, the Points b and p being infinitely near one another; it may be proved, after the very same Manner as above, that $\frac{Spq \times bdq}{br}$ is $= \frac{2SGq \times SAq}{Sp}$. But (by 36 *Prop.*) the centripetal Force in P is to the centripetal Force in p, as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$; *that is* (when the said Force

is directed to the Center of the Ellipse) as $\frac{2SGq \times SAq}{Sp}$ is to $\frac{2SGq \times SAq}{SP}$, or as $\frac{1}{Sp}$ is to $\frac{1}{SP}$, because $2\,SG_q \times SA_q$ is a given or constant Quantity. Therefore the centripetal Force in P is to the centripetal Force in p (as $\frac{1}{Sp}$ is to $\frac{1}{SP}$ or) as SP is to sp the Distances directly; because Fractions of the same Numerator, are reciprocally proportional to their Denominators.

Proposition XL.

Lem. *Fig.* 40.

LET BGA *be an Ellipse,* BA *its longer Ax,* C *the Center,* CG *half the shorter Ax, and* F *one of the Foci; then, if from* F *there be drawn an Ordinate* FE *to the longer Ax* BA, *I say that* FE *is more than* FA.

Put BC or CA $= a$, CG $= c$, CF $= e$; then is BF $= a + e$.

By 4 *Cor.* 2. 4. *Miln. Conic.* FE is equal to half the Parameter of the Ax

AB;

A B. And (by 4 *Cor.* 24. 1. *Miln. Conic.*) F E $\times$ C A or F E $\times a$ is $=$ C G$_q$ $= c^2$. Whence F E is $= \frac{c^2}{a}$. Alſo (by 2 *Cor.* 20. 1. *Miln. Conic.*) as B F $\times$ F A : B C $\times$ C A :: F E$_q$: C G$_q$, *that is*, as $\overline{a+e} \times$ F A : a^2 :: $\frac{c^4}{a^2}$: c^2. Whence there is $\overline{ac^2+ec^2} \times$ F A $=$ ($a^2 \times \frac{c^4}{a^2} =$) c^4 : And conſequently F A $=$ ($\frac{c^4}{ac^2+ec^2} =$) $\frac{c^2}{a+e}$. But, as before, F E is $= \frac{c^2}{a}$; and 'tis evident that $\frac{c^2}{a}$ is more than $\frac{c^2}{a+e}$. Therefore is F E more than F A. W. W. D.

Corollaries.

1. THE Semi-parameter or Ordinate F E drawn from the Focus F, is longer than any right Line F D drawn from the ſaid Focus to any Point D of the elliptick Curve between A and E. This will be evident by deſcribing a Circle from F as a Center at the Diſtance F E.

2. It is evident, that any right Line F K drawn from the Focus F to any Point of

of the elliptick Curve between F and B, is longer than FE and shorter than FB:

3. It is evident, that the Line FI lying nearer FE, is longer than FD lying further from FE. And that any Line FK lying nearer FB, is longer than FG lying further from FB.

4. It is evident, that FA the Part of the longer Ax lying between the Focus F and the nearest Vertex A, is the shortest of all right Lines that can be drawn from the said Focus to any Point of the elliptick Curve: And that the remaining Part FB, is the longest of all.

PROPOSITION XLI.

Lem. Fig. 40.

THE Distance FG *between either Focus* F *of an Ellipse and the End* G *of the shorter Ax, is a mean arithmetical Proportional between the Segments* BF *and* FA *of the longer Ax, made by the same Focus* F.

For

For PG being drawn from the other Focus P, there is (by 5. 4. *Miln. Conic.*) FG + PG or 2 FG = AB; and consequently FG = $\frac{1}{2}$ AB or the half Sum of BF and FA: Wherefore FG is a mean arithmetical Proportional between BF and FA. W. W. D.

PROPOSITION XLII.

Theor. *Fig.* 41.

IF a Body move in an Ellipse by a projectile Force, and a centripetal Force constantly tending to a Focus of the Ellipse: Its Velocity in any Point in its second Revolution, will be the same it was in that Point in its first Revolution: And so in any other Revolution.

Let S be the Center of a centripetal Force which joined with a projectile, makes a Body describe the Ellipse AKPN whose longer Ax is AP and shorter BF; suppose also S to be one Focus of the Ellipse. Then (by 32 *Prop.*) the Body will

will deſcribe Areas proportional to the Times of Deſcribing. Let the infinitely little Areas ASC, CSD, DSE, ESH, HSI, &c. be equal; and conſequently the Times in which the Body deſcribes the infinitely ſmall Parts AC, CD, DE, EH, HI, &c. of the Ellipſe, will be equal. Therefore (by 1 *Prop.*) the Velocities of the Body in theſe Parts will be in the ſame Proportion as the ſaid Parts; for the Motions in theſe infinitely little Parts may every one be conſidered as uniform: And ſo the Velocity in AC will be as AC, the Velocity in CD (though greater than that in AC) will be as CD, the Velocity in DE as DE, and ſo on. But it is plain that AT is = AC, TR = CD, RN = DE, and ſo on; and conſequently the Velocities in theſe Parts are the ſame reſpectively. From whence it is evident, that the Velocity in A in the ſecond Revolution, is the ſame it was in A in the firſt Revolution; and ſo in any other Point of the Ellipſe. What is proved of a ſecond Revolution, holds good, after the very

ſame

same Manner, in a third, fourth, *&c.* W. W. D.

Corollaries.

1. FROM this *Prop.* and 40 *Prop.* with its *Corollaries* it is evident, that the Body's Velocity (the Center of the centripetal Force being the Focus S of the Ellipse) is still increasing from A through K to P, and again decreasing from P through B to A; so that the Velocity in A is least, and in P greatest. It is also evident from 41 *Prop.* that the Velocity in B and F, the Ends of the shorter Ax, is a Mean between the least Velocity in A and the greatest in P.

2. Hence it is plain, that all that Mr. *Gordon* advances in *Pag.* 87, 88, 89 of his *Remarks*, concerning the constant Increase of Velocity of a Body revolving in an Ellipse, about one of the Foci as the Center of the centripetal Force, is false, and his Banter groundless. For from what we have just now proved it is evident, that though the Velocity increases in one

Half

Half of the Ellipſe, it gradually decreaſes as much in the other; and that, after a complete Revolution, the Velocity is the ſame it was at firſt.

Proposition XLIII.

Lem. *Fig.* 42.

THE *verſed Sine of an indefinitely ſmall Arch of a Circle, is equal (at leaſt extremely near) to the Square of the ſaid Arch divided by the Diameter :* That is, *if* AB *be a very ſmall Arch of a Circle whoſe Diameter is* AG, *the verſed Sine* AC *is (very nearly) equal to the Square of the Arch* AB *divided by the Diameter* AG.

This already has been virtually proved in 35 *Prop.* but we ſhall here explicitely demonſtrate it.

Draw the Chord AB, the Sine CB, and the Line GB. Then (by 2 *Cor.* 8. 6. *Eucl.*) as is AG : AB : : AB : AC; whence AC is $=\frac{ABq}{AG}$, *that is*, the verſed Sine AC of the Arch AB is equal to the Square of

of the Chord A B divided by the Diameter A G. But an indefinitely ſmall Arch and its Chord coincide. Therefore the verſed Sine A C, is equal to the Square of the Arch A B divided by the Diameter A G. W. W. D.

A Scholy.

MR. *Gordon* in his *Remarks* begins at the Foot of *Pag.* 38. to quarrel hard with Sir *Iſaac Newton* and Dr. *Gregory*, for their affirming that the Force by which the Moon is hindred from running out in ſtraight Lines and kept in her Orbit, is the ſame with that Force by which heavy Things fall to the Ground; and endeavours to confute them. In order to which he affirms in *Pag.* 42. that it very clearly appears, that the real Motion of the Moon, is (by *Newton*) compared with the apparent Part of falling Bodies only. Now ſuppoſe this be granted him; though I ſee no good Reaſon that it ſhould, ſince the Fall of a Body very near the Surface of the Earth, is

M found

found by many Experiments to be about $15\frac{1}{12}$ *Paris* Feet, in a Second of Time, in a free Space; so that whether the Earth roll about its Axis or not, the Fall of $15\frac{1}{12}$ such Feet in a Second of Time is determined by Mr. *Hugens*, Sir *Is. Newton*, and their Followers, to be the full and real (and not apparent) Effect of Gravity near the Earth: Yet the odds of falling in a Second, a Space more than $15\frac{1}{12}$ Feet, *viz.* such a Space more as is equal to the versed Sine of 17″ (as *Gordon* would have it, or rather 15″, so much as any Point of the Earth's Surface runs, by its diurnal Motion, in a Second of Time) of the Earth's Circumference, besides the said $15\frac{1}{12}$ Feet or 181 Inches, which at most is but $\frac{3}{4}$ of an Inch, is so small in respect of 181 Inches, that in such nice Experiments it need not be considered.

That

That the versed Sine of an Arch of $15''$ of a great Circle on the Earth is not full $\frac{3}{4}$ of an Inch, will thus appear. Such a great Circle's Periphery is, by late accurate Observations, found to be 123249600 *Paris* Feet, and consequently the Diameter to be 39231600 such Feet: Now as 1296000 the Seconds in the whole Periphery is to 123249600 the Feet in the whole Periphery, so is 15 Seconds of the Periphery to 1426'5 Feet, answering to 15 Seconds. This Arch 1426'5 Feet squared, and its Square 2034902'25 divided by the Diameter, the Quote '052 will (by 43 *Prop.*) give the versed Sine of $15''$ sought: And this Decimal '052 of a Foot is not full $\frac{3}{4}$ of an Inch. From whence it is plain, that the *Remarker* here makes a Noise to little Purpose, especially since the mean Diameter of the Moon's Orbit is not yet completely determined, though it be determined to be about 60 Diameters of the Earth.

PROPOSITION XLIV.

Theor.

THE Force whereby the Moon tends to the Center of the Earth, and is kept in her Orbit; is the same with the Force of Gravity, whereby terrestrial Bodies tend to the said Center.

The Time of the Moon's Revol[illegible] in her Orbit is 27 Days, 7 Hours, 43 Minutes, or 2360580 Seconds of Time. In 360° there are 129600″.

And as 2360580 Seconds of Time is to 1296000 Seconds in the Periphery, so is 1 Second of Time to $\frac{129600}{236058}$ of a Second in the Periphery, equal to ,549017 of a Second, equal to the Arch of the Moon's Orbit that is described in 1 Second of Time.

The Moon's Orbit is 7394976000 *Paris* Feet; and the Diameter of her Orbit is 2353896000 *Paris* Feet.

Then as 1296000, the Seconds in the whole Periphery, is to 7394976000, the Feet

Feet in the whole Periphery, or as 1296 is to 7394976, so is '549017″ of the Periphery to 3132'691 Feet, answering to '549017″. So then the Arch of the Moon's Orbit, run in 1 Second of Time, is 3132'691 Feet.

This Arch of 3132'691 Feet being squared, and the Square 9813752'901481 divided by the Diameter 2353896000, the Quotient '004169 of a Foot will (by 43 *Prop.*) give the versed Sine of the Arch the Moon runs in a Second, which is the Measure (at least extremely near) of the Moon's centripetal Force in her Orbit: But the said Force is as the Square of the Distance reciprocally. Therefore, if the Moon was brought to the Surface of the Earth, or 60 Times nearer the Center than she is, her centripetal Force would be 60×60, or 3600 Times '004169, of a Foot, or 15'0084 Feet equivalent to 180'1 Inches. So then the Moon near the Surface of the Earth being let fall, would descend by her centripetal Force in a Second of Time, 180'1 Inches. But it is known by many Experiments, that

heavy Bodies near the Surface of the Earth fall in a Second of Time 15 $\frac{1}{12}$ Feet or 181 Inches: And these Numbers 180'1 and 181 differ but very little from one another. Therefore the Moon's centripetal Force is all one with Gravitation, or that Force whereby heavy Bodies near the Surface of the Earth tend to the Earth's Center.

This Calcul is founded upon the Supposition, that the Diameter of the Moon's Orbit is 60 Diameters of the Earth: But if we suppose the Diameter of the Moon's Orbit to be somewhat more (as probably it is) *viz.* 60 $\frac{1}{4}$ Diameters of the Earth, and renew the Calcul, we will find that the Moon near the Surface of the Earth, would descend by her centripetal Force about 182 Inches, in a Second of Time: And this will answer all that Mr. *Gordon* can demand about his real and apparent Gravity, as is plain from *Sch.* 43 *Prop.*

A Scholy. *Fig.* 43.

MR. *Gordon* in *Pag.* 52 of his *Remarks* delivers a *Theorem*, from which in *Pag.* 55 he infers a *Corollary*, design'd not only to overturn the preceeding *Proposition* about the Gravitation of the Moon, but also all Measures and Estimates of centripetal Forces by versed Sines. His *Theorem*, in his own Words, is as follows.

A Body describing an Arch a d, *really falls from the Tangent of every Point of that Arch a certain Space, and the Sum of all those Spaces is, in respect of the verse Sine of* a d, *infinitely little.*

This *Theorem* looks very like a Paradox, and yet if the Arch *a d* be supposed infinitely little, it will perhaps be found to agree better with a Principle delivered by Sir *Is. Newton*, Dr. *Gregory*, and Mr. *Whiston*, than any Thing in all Mr. *Gordon*'s *Remarks*. The said Principle is *Newton*'s 1 *Cor.* 11 *Lem.* 1 *Lib. Princip.* *Gregory*'s *Schol.* 24 *Prop.* 1 *Lib. Astron.* and

and *Whiston's* 4 *Cor.* 2 *Prop. Prælect. Physico-mathem.* which we have before laid down in *Cor.* 35 *Prop. Gordon's* Demonstration of his *Theorem* being all bare Assertion, without any Reasons assigned, I shall not be at the Pains to examine it; but shall only more fully express the true Meaning of his *Theorem*, and after I have endeavoured to give a more clear, succinct, and accurate Demonstration, deliver his *Corollary*: Which done, I believe it will easily appear, that the said *Corollary* has either no Connection at all with the said *Theorem*, or else that the Connection is so obscure, that the *Remarker* is oblig'd to shew it. The true Meaning then of the *Theorem* I take to be this; *Suppose an infinitely little Arch* a d *of a Curve to be divided into an infinite Number of Parts, and Tangents drawn to all the Points of Division; a centripetal Force constantly directed to a certain Point* s, *will make a Body describing the Arch* a d *fall from all the Tangents certain Spaces, and the Sum of all these Spaces together will be infinite-*

ly

ly little in respect of k d *the versed Sine of the Arch* a d, *or right Line drawn parallel to* s a *from* d *to* a l *the Tangent in* a. Which I thus demonstrate.

Suppose the infinitely little Arch *a d* divided into two equal Parts *a c*, *c d*: When the Body has come from *a* to *c*, the centripetal Force tending to s has made it fall from the Tangent of the Point *a* the Space *e c*, and when it is come to *d*, the said Force has made it fall from the Tangent of the Point *c* another Space equal to *e c*, at least infinitely near so; the Sum of which two Spaces, *viz.* 2 *e c* is $= \frac{1}{2}$ *k d*, because (by 35 *Prop.* and its *Cor.*) as is *e c* : *k d* :: $a c_q$: $a d_q$ :: 1 : 4. If, again, we suppose the Arch *a d* divided into three equal Parts *a b*, *b c*, *c d*, the Body, by the Force tending to s, will fall from the Tangent of *a* the Space *f b*, from the Tangent of *b* as much, and from the Tangent of *c* also as much, *that is*, it will fall from the three Tangents of *a*, *b*, *c*, three Spaces, every one of which is equal to *f b*; the Sum

Sum whereof, *viz.* $3fb$ is $= \frac{1}{3} kd$, because (by *Cor.* 35 *Prop.*) as is $fb : kd :: abq : adq :: 1^2 : 3^2 :: 1 : 9$. In like Manner, if we suppose the Arch ad divided into four equal Parts, the Sum of the Spaces fallen from the Tangents of the Point a and the next Three Points of Division, will be equal to $\frac{1}{4} kd$: And so forth. Therefore, since the Sum of the Spaces fallen from the Tangents constantly decreases, as the Number of the Points of Contact increases; it is evident, that, when the Points of Contact are infinite in Number, the Sum of all the Spaces fallen from the Tangents, while the Body is describing the Arch ad, is infinitely little in respect of the versed Sine kd of that Arch. W. W. D.

The *Corollary* our *Remarker* pretends to draw from the foresaid *Theorem*, is what follows.

Hence it appears, that all Estimates of the Quantity of a Force turning a Body from straight Lines into a Curve, if they measure the Force that produces any assign-

ſignable Arch by the verſe Sine of that Arch, or compare the Forces that produce any two Arches, one whereof is greater than another, by the verſe Sine of thoſe Arches, are erroneous.

We ſhall leave the *Remarker* to make good his *Corollary*, either from his *Theorem*, or any other Way he ſhall think fit. In the mean Time, whatever becomes of his *Theorem*, and whether his *Corollary* be by a juſt Conſequence deducible from his *Theorem* or not, I think we can prove his *Corollary* to be falſe. For ſuppoſe *a d* and *d z* to be infinitely ſmall Particles of the Curve *a d z*, whether equal or not, deſcribed by a Body in very ſmall, but equal Particles of Time: Let *a l* be a Tangent at *a*, and *d n* a Tangent at *d*; alſo let *d k* be parallel to *s a*, and *z n* parallel to *s d*: While the Body is moving from *a* to *d*, in the infinitely little or naſcent Arch *a d*, the centripetal Force tending to s is infinitely little altered, which is much the ſame Thing as to ſay, it is not altered at all, but continues the ſame in every Point of the

the Arch *a d*, till it come to *d*. In like Manner, while the Body is moving from *d* to *z*, in the infinitely little Arch *d z*, though the centripetal Force in *d z* may be ſomewhat leſs or more than it was in *a d*, yet it muſt be ſuppoſed to continue the ſame unaltered thro' whole *d z*, till it come to *z*. Now *k d* is the Effect of the centripetal Force, when the Body has moved from *a* to *d*, and *n z* the Effect of the ſaid Force, when the Body has moved, in an equal Particle of Time, from *d* to *z*; and the ſaid Forces are the ſole and adequate Cauſes of the ſaid Effects: Therefore the infinitely little or naſcent Lines *k d* and *n z*, muſt (by 2 *Max.*) be proportional (at leaſt infinitely near) to the centripetal Forces in the Points *a* and *d*, or in the Arches *a d* and *d z*. Therefore *k d* is the (infinitely near) Meaſure or Eſtimate of the Quantity of the centripetal Force at *a*, and *n z* the Meaſure of the ſaid Force at *n*; notwithſtanding that the preceeding *Corollary* affirms the contrary, by pronouncing all

Eſti-

Estimates of centripetal Forces by versed Sines erroneous.

Note, That it is not necessary that the Arches *ad* and *dz* should be contiguous, although they be so in the present Figure.

Again suppose *ad* to be an infinitely small Arch of the Curve *adz*, and consequently the centripetal Force to continue the same from *a* to *d*; suppose also *ab* to be a Part of the Arch *ad*: Then, notwithstanding the preceeding *Corollary*, *fb* is the Measure of the centripetal Force in *a*, with respect to the Time wherein the Arch *ab* is described; and *kd* the Measure of the same Force in *a*, with respect to the Time wherein the Arch *ad* is described.

Note, that, since *ad* is supposed an infinitely little or nascent Arch, it is all one upon the matter, whether the infinitely little or nascent Line *kd* be perfectly and rigorously parallel to the right Line *sa*, or be a Particle of the right Line *sd* produced, and consequently infinitely near parallel to *sa*. And, in like manner, it is

all one whether the nascent Line *nz* be perfectly parallel to *sd*, or be a Particle of *sz* produced.

It will not, I think, be improper in this Place to observe, that though the projectile and centripetal Forces in a Body moving in a Curve, may be very unequal; yet the centripetal and centrifugal Forces (taken in a strict and rigorous Sense, the former as urging directly to a Center, and the latter directly from it) are in every Point of the Curve exactly equal. For suppose, as before, *ad* to be an infinitely small or nascent Particle of the Curve *adz*, *al* a Tangent in the Point *a*, s the Center of a centripetal Force, which joined with a Projectile Force, in the Direction of the Tangent *al*, obliges a Body to describe the Curve *adz*: Draw the right Lines s*a* and s*dk*, and *dk* will be infinitely near parallel to s *a*. Then, as above, the nascent Line *kd* will be the full Effect of the centripetal Force, while the Body is moving in the Curve from *a* to *d*, and also the Measure of the same in the

Point

Point *a*. But while the Body is urged by the centripetal Force directly towards the Center s, though, by reason of the projectile Force conjoined, it cannot really move directly towards the Center, but must move in the Curve; the Body's natural Tendency being (by 1 *Max.*) according to the Direction of the Tangent *a l*, if (the projectile or natural Force remaining) the centripetal Force in *a* was destroyed, the Body would come to *k*, in the same Instant of Time, that it came to *d* by the Action of the centripetal Force, and consequently would be removed from the Curve, the nascent Space *d* k. Now the Force that would cause the Body to move the Space *d k* from the Curve, in the Directon of the Line s *d k*, *viz.* from *d* to *k*, in the same Particle of Time, that the Body really moves in the Curve from *a* to *d*, is what is properly called *the centrifugal Force*, which has its Origin indeed from the projectile or tangential Force. Therefore, since the Line *d k* would be the true and full Effect of the centrifugal

Force, while the Body is moving from *a* to *d*, and consequently the Estimate and Measure of the said Force in *a*; as it really is the true and full Effect of the centripetal Force, and Measure of the same in *a*; it is evident, that the centrifugal and centripetal Forces, in the said Point *a*, are equal. And so in any other Point of the Curve.

PROPOSITION XLV.

Theor.

EVERY Body A *which, a Radius being drawn to the Center of another Body* B *howsoever moved, describes Areas, about that Center, proportional to the Times of describing; is urged by a Force compounded of the centripetal Force tending to this second Body* B, *and of all the accelerating Force whereby the said Body* B *is urged:* That is, *the first Body* A *will, at the same Time, be urged by the said two Forces jointly.*

This

This seems pretty evident, for else the Proportionality of the Areas to the Times could not be constantly observed, which would overturn the *Hypothesis*.

Otherwise thus: If both Bodies be urged in parallel Lines, by a new accelerating Force equal and contrary to that whereby the second Body B is urged; all the accelerating Force in B, and as much accelerating Force in A, will be destroy'd: Yet the Body A will (by 1 *Sch.* 33 *Prop.*) continue to move with its remaining Force, after the same manner, as at first, in respect of B, and so will describe about B Areas proportional to the Times. Then, since A, by its remaining Force, describes round B, Areas proportional to the Times, the said Force in A (by 33 *Prop.*) tends to B. But when at first A described about B Areas proportional to the Times, A was urged by the accelerating Force that was destroyed in it, or all the accelerating Force in B, and the remaining Force in A. Therefore A is urged not only by the said remaining Force, or Force tending

to B, but also by all the accelerating Force whereby B is urged. W. W. D.

A Scholy.

OUR *Remarker*, in *Pag.* 35, 36, 37, 38, runs out at a strange Rate, quibbling against this *Proposition* to no Purpose, but speaking with a mighty Air of Assurance and Vanity; and feigns, at the End of *Pag.* 37 and Beginning of *Pag.* 38, a gross Absurdity he pretends will follow from the said *Proposition*, from our Authors (Sir *If. Newton* and Dr. *Gregory*) taking very good Care, as he says, not to mention that Half of the Cause, which consists in the Endeavour of the Body to run out in Tangents; which (continues he) if they had not done, the Absurdity of supposing the Cause of any Body's Motion to be an Endeavour in the Body, at every Instant of Time, to run out in two different Lines (as the Tangents of those two different Curves are) at the same Time, is so very gross and evident, that it could never have been

been ſwallowed by any Man of common Senſe.

But here our *Remarker* ſeems to talk after ſo ridiculous and extravagant a Manner, that one would be tempted to think, that either he at this Time was not in his right Senſes, or elſe that he was reſolved to ſay Something or other, right or wrong, Senſe or Nonſenſe, againſt the ſaid *Propoſition* and its Authors, rather than let it paſs for Truth, and allow them to be in the Right. For what Occaſion was there, I would fain know, to mention, in the preceeding *Propoſition*, the projectile or tangential Force of the Body A, though it is not excluded, ſince the Body A (by 33 *Prop.*) is obliged to move, as the laſt *Propoſition* expreſſes, by a projectile Force, and a centripetal Force tending to the Body B. Suppoſe the Body A had in that *Propoſition* been ſaid to deſcribe about B, Areas proportional to the Times by a projectile Force once impreſt, and a conſtant centripetal Force tending to B; would that Expreſſion have altered the Senſe of the *Propoſition*?

no,

no, it would not in the least have affected it, nor its Demonstration neither. The said *Proposition* is also as little or less concerned to mention a tangential Force in the Body B, because it may or may not have one.

PROPOSITION XLVI.

Theor. *Fig.* 44.

THE *Force or Efficacy of a Vertue which is propagated to or from a Center in streight Lines, every way round in a circular Space, is at different Distances from the Center, as the said Distances reciprocally. And the Force or Efficacy of a Vertue which is propagated to or from a center in streight Lines, every way round in a spherick Space, is, at different Distances from the Center, reciprocally as the Squares of the said Distances.*

1. Let S be the Center, to or from which the Vertue is propagated in a circular Space, about which describe two circular Peripheries *c d m*, *e f n*, at any

Distances sd, sf: I say, that the Force or Efficacy of the Vertue, at the Distance sf, will be to the Force of the same, at the Distance sd, reciprocally as sd is to sf.

For the same Quantity of the Vertue that is equally diffused through the Arch cd, is also equally diffused through the similar Arch ef: Then if ef be double of cd, or eh (the Half of ef) equal to cd, the Quantity of the Vertue in eh will be just half the Quantity of it in cd; and so the Force of the Vertue in ef will be half its Force in cd; *that is*, the Force in ef will be to the Force in cd, as cd to ef, or as sd to sf, for similar Arches of Circles are as the Semi-Diameters. In like manner, if ef be $= 3\,cd$, the Efficacy of the Vertue diffused thro ef, will be only $\frac{1}{3}$ of the Efficacy thereof diffused thro' cd, because the Vertue in ef will be three Times sparser than in cd; consequently its Efficacy or Force in ef will be to its Efficacy in cd, as cd to ef, or as sd to sf. And so universally, whatever Proportion ef bear to cd, the

Efficacy

Efficacy of the Vertue in *ef* is to its Efficacy in *cd*, reciprocally as the Distance *sd* is to the Distance *sf*.

2. Suppose the Vertue be diffused or propagated in streight Lines from the Center *s* every way round in a Sphere, about which Center describe two spherick Surfaces *cdm*, *efn*: I say, that the Efficacy of the Vertue, at the Distance *sf*, will be to the Efficacy of the same, at the Distance *sd*, reciprocally as *sdq* to *sfq*.

Let *ef* and *cd* represent like Parts of the spherick Surfaces *efn* and *cdm*; and so there is as *ef* : *cd* :: *efn* : *cdm*. Now we can prove, just as in the first Part, that the Efficacy of the Vertue in the Surface *ef* is to its Efficacy in the Surface *cd*, reciprocally as the Surface *cd* is to the Surface *ef*, or as the whole spherick Surface *cdm* is to the whole spherick Surface *efn*. But, by *Archimedes*'s Doctrine of the *Sphere* and *Cylinder*, spherick Surfaces are as the Squares of the Semi-diameters; and consequently the spherick Surface *cdm* is to the spherick.

rick Surface efn, and so the Surface cd to the Surface ef, as sdq to sfq. Therefore the Efficacy of the Vertue in the Surface ef, at the Distance sf, is to its Efficacy in the Surface cd, at the Distance sd, reciprocally as sdq is to sfq.

A Scholy.

BY the preceeding Demonstration it is solidly proved, that the Efficacy of a Vertue diffused to or from a Center, in streight Lines, in a circular Space, and acting upon an Arch, is reciprocally as the Distance of the Arch from the Center; and that the Efficacy of a Vertue diffused in streight Lines to or from a Center in a Sphere, and acting upon a spherick Surface, is reciprocally as the Square of the Distance of that Surface from the Center. From whence Mr. *Gordon* pretends by a Parity of Reason to prove, in his third *Theorem Pag.* 75, that the Force or Efficacy of any Vertue, that spreads itself in streight Lines through all the surrounding Space, equally to or from a Center, and

acts

acts upon the ſolid Content or trine Dimenſion of Bodies, muſt, in different Diſtances ftom that Center, be as the Cubes of thoſe Diſtances (we muſt ſuppoſe, though he does not expreſs ſo much, that he means) reciprocally.

Now in order, to prove that his ſaid third *Theorem* is falſe and inconſiſtent with *Reaſon*, and conſequently that his pretended parity of Reaſon (on which alone it is founded) does here quite fail him; let the following *Principle* be carefully obſerved, *viz.* that if the ſame Quantity of a Vertue be diffuſed through a greater Space and a leſſer, whether theſe Spaces be both linear, or both ſuperficial, or both ſolid, the Efficacy or Force of the Vertue in the greater Space will be leſs than in the leſſer Space (becauſe the Vertue is more ſparſe, being more ſcattered, in the greater than in the leſſer Space) and that in the ſame Proportion that the leſſer Space bears to the greater; that is, *the Efficacy of the Vertue in the greater Space will be to its Efficacy in the leſſer, reciprocally as the leſſer*

Space

Space is to the greater. So the Force or Efficacy of a Vertue diffused thro' a double Space, will only be half the Force of the same Quantity of the Vertue diffused thro' the single Space; because the Vertue will be twice as sparse in the double Space as it is in the single Space. In like manner, the Efficacy in a triple Space, will be a third Part of the Efficacy of the same Quantity in the single Space; because the Vertue will be three Times sparser in the triple Space than in the single.

Suppose now two similar Bodies or Solids, arising from the Revolution of the plane Surfaces *c r t d*, *a e f b* (in *Fig.* 44.) about the Radius *s k*, whose respective Distances from the Center *s* let be *s t*, *s f*. Then these similar Solids will be in a triplicate Proportion of the Arches *r t*, *e f*, or the Lines *t d*, *f b*, their homologous Sides. Let both Solids be conceiv'd as made up of an indefinite Number of concentrick spherick Surfaces (not mathematical but physical) of indefinitely small but equal Thickness: Then, if the Arch

r t be as 1, and the Arch *e f* as 2; and consequently also *t d* (the Thickness of the Solid *c r t d*) as 1, and *f b* (the Thickness of the Solid *a e f b*) as 2; the Solid *c r t d* will be as 1, and the Solid *a e f b* as 8: And there will be twice the Number of spherick Surfaces in the Solid *a e f b*, that there are in the Solid *c r t d*, because *f b* is = 2 *t d*. Now the Quantity of the Vertue, propagated in right Lines around alike, thro the spherick Space, to or from the Center s, is the same in every one of the said spherick Surfaces *r t*, *c d*, *e f*, *a b*, and all the rest of the intermediate ones: Therefore the Quantity of the said Vertue diffused thro' the Solid *a e f b*, is double of the Quantity diffused thro' the Solid *c r t d*. But the Solid *a e f b* is 8 Times bigger than the Solid *c r t d*, as before. Therefore, twice the Quantity of the Vertue being diffused thro' 8 Times the Space, or (which is all one) the same Quantity of it being diffused thro' 4 Times the Space; the Efficacy of the Vertue in the Solid *a e f b*, is (by the *Principle* laid

down

down before) to the Efficacy of it in the Solid *crtd*, reciprocally as the Solid *crtd* is to 4 Times the ſame Solid, or half the Solid *aefb*, *that is*, as 1 is to 4: But ſince, as before, the Arch *rt* is $=1$; and the ſimilar Arch $ef = 2$: Therefore is $st : sf :: 1 : 2$, and conſequently $st_q : sf_q :: 1 : 4$. And ſince the Solid *crtd* is to half the Solid *aefb* as 1 is to 4, or as st_q is to sf_q. Therefore the Efficacy of the Vertue in the Solid *aefb*, is to the Efficacy of it in the Solid *crtd* as st_q to sf_q, *that is*, reciprocally as the Squares of the Diſtances of the ſaid Solids from the Center *s*.

If the Arches *rt* and *ef* be as 1 and 3; and conſequently alſo *td* and *fb* as 1 and 3; the ſimilar Solids *crtd* and *aefb* will be as 1 and 27, the Cubes of 1 and 3; and there will be thrice the Number of ſpherick Surfaces in the Solid *aefb*, that there are in the Solid *crtd*; and conſequently thrice the Quantity of the Vertue will be diffuſed thro' the Solid *aefb* that is diffuſed thro' the Solid *crtd*, or the ſame Quantity of the Vertue will be

diffused thro' a third Part of the Solid *aefb*, or 9 Times the Solid *crtd*, that is diffused thro' the single Solid *crtd*. Therefore the same Quantity of the Vertue being diffused thro' 9 Times the Space, the Efficacy of the Vertue in the Solid *aefb* will be to its Efficacy in the Solid *crtd*, reciprocally as the Solid *crtd* is to 9 Times the Solid *crtd*, or as 1 to 9. But, as above, the Arches *rt* and *ef* are as 1 and 3; therefore is $st : sf :: 1 : 3$, and consequently $st_q : sf_q :: 1 : 9$. Therefore the Efficacy of the Vertue in the Solid *aefb* is to its Efficacy in the Solid *crtd* as st_q to sf_q, *that is yet*, reciprocally as the Squares of the Distances of the said Solids from the Center s. And, in like manner, in any other Proportion of *rt* to *ef*, or of *st* to *sf*.

But, after all, it is not necessary to prove, that a Vertue, such as a centripetal Force, propagated in right Lines around in a Sphere, acting upon the trine Dimension or Substance of a Body, is at different Distances of the Body from the

the Center reciprocally as the Squares of the Diſtances; ſince the Diſtance of a Body from the Center of a Sphere, is as various as the Diſtances of the ſeveral Particles, of which the Body is compoſed, are, which are innumerable; though the Diſtances of all the Particles of a ſpherick Surface from the Center, is every where the ſame. So then, though the centripetal Force acts on the Subſtance and internal Parts of Body, or (which is all one) on all the phyſical Surfaces of which the Body is compos'd, from its inmoſt to its outmoſt Extremity, and not on its external Surface only (as Mr. *Gordon* would have it) yet it cannot act alike on every Part of the Body, but muſt (by 2 *Part* 46 *Prop.*) ſo act, that its Efficacy muſt ſtill be reciprocally as the Square of the different Diſtance of the Parts from the Center.

Proposition XLVII.

Theor. *Fig.* 45.

THE *Flux and Reflux of the Sea proceed from the Attractions, or centripetal Forces of the Sun and Moon, but especially of the Moon.*

This may be proved in respect of the Moon, thus. Let M represent the Moon; ZENF the Earth, C its Center, Z the Place where the Moon is in the Zenith, N where in the Nadir, EF the Horizon. Now 'tis evident, that the Water in Z, being nearer the Moon M, is more attracted by her, than the Center of the Earth C (or any Parts about the Horizon EF) and this again more than the Water in N. So then the greatest Attraction of the Parts of the Earth towards the Moon is at Z, the least at N, and the mean Attraction at C or EF; *that is*, since C tends faster towards M than N does, and Z faster than C, we may consider the Attraction at C as none (or at least almost none) at all, and the

At-

Attractions at Z and N as tending contrary ways, *viz.* the Attraction at Z tending directly to the Moon, and that at N directly from her; that is yet the same Thing, as if the Parts about C were at Rest, and those about Z and N were moving contrary ways. From all which it will be evident, that the Water of the Earth about the Horizon E F will gravitate and be pressed more strongly towards the Center C, than the Water about Z and N where the Moon is vertical. And hence it plainly appears, that the Water at Z and N will rise and swell, and that the Water about E F will settle down and run towards Z and N; and so the Earth will put on an oval Figure. This certainly would be the Consequence, if the Earth was covered round with Water to any considerable Depth: And even in its present State, it will imitate that Form as much as the dry Land will allow, and by the Moon's Motion occasion the Ebbing and Flowing of the Sea.

The like is to be understood of the Sun's Attraction, though in a much smal-

ler Degree, upon account of his vast Distance.

Scholy I. *Fig.* 45.

IT will not be amiss here, to assign the Proportion that there is between the Difference of the *Moon's* Attractions at Z and C, and the Difference of her Attractions at C and N; which is pretty easily done thus. Let the Attraction of the Earth's Center C, towards the Moon M, be 3600, the Square of 60 Semi-diameters of the Earth, or the Moon's middle Distance from the Center of the Earth; for that Attraction may be expressed by any Number we please: Then, since the Moon's Attractions, at different Distances, are reciprocally proportional to the Squares of the Distances, her Attractions at Z, C, N must be (as MN *q*, MC *q*, MZ *q*, or as 61 × 61, 60 × 60, 59 × 59, or) as 3721, 3600, 3481 in Order: Therefore the Difference of the Attractions at Z and C is 121, and the Difference at C and N is 119, which two Differences are pretty near

near equal; so that the Water at Z can be but insensibly higher than the Water at N.

A Corollary.

FROM hence it is plain, that Mr. *Gordon's* 4th *Theorem*, in *Pag.* 105 and 106 of his *Remarks*, is false; wherein he pretends to prove, that the Protuberance of Water which is under the Moon, is considerably higher than the opposite Protuberance, *that is*, that the Water at Z (in our *Fig.* 45.) is considerably higher than that at N.

The Substance of his Demonstration is as follows. The Water at N is pressed towards the Earth's Center C by the strongest Attraction, *viz.* the Attraction of the Earth and the Attraction of the Moon M together; whereas the Water at Z is pressed to the Earth's Center by the least Attraction, *viz.* the Attraction of the Earth diminished by the Attraction of the Moon, and so gravitates less than the Water

Water at N. The Waters alſo at E and F are drawn towards Z, and not towards N; and the Water at N, being moſt preſſed, will run towards theſe low Places E and F; but the Water at Z, being leaſt preſſed, will not. So that the Water at N, E, and F will run towards Z, and make a conſiderably greater Protuberance at Z than at N.

The Fault of this Demonſtration in ſhort is, that though it conſiders the Attractions or Tendencies of the Parts of the Earth at Z, N, E, and F towards the Moon, yet it does not compare theſe Attractions together; and (which is the greateſt Defect) entirely neglects the Tendency of the Center C and Parts about it towards the Moon.

Scholy. II.

BY this Time, I hope, the Reader ſees, that I have demoliſhed all *Gordon's* main Forts, from which he has endeavoured to deſtroy the *Newtonian Philoſophy*: And as to his ſmaller Batteries which at

laſt

laſt he runs to, they are of ſo ſmall Moment, that the leaſt Attack in the World will ſoon ruine them. Theſe laſt are chiefly the Abſurdities, he thinks will follow from the Laws of univerſal Gravitation, and the Reſiſtence the celeſtial Bodies meet with from the Fluid of Light ſcattered through the celeſtial Regions. As to the Firſt of theſe, I ſuppoſe it will eaſily be allowed, that an omnipotent Being can impoſe any Laws upon Matter, that are not inconſiſtent with its Nature, and involve no Contradiction, as the univerſal Law of Gravitation, 'tis plain, does not. So that all the Abſurdities our *Remarker* deduces from hence, are mere Roveries of his own Brain, without any real Foundation. And as to the Second, the Rays of Light are ſo extremely fine, and ſo ſcattered, that the Quantity of the whole Fluid of Light bears hardly any Proportion at all, to the Quantity of the immenſe Space through which this Fluid is diffuſed; and conſequently the Reſiſtence that the Planets and other celeſtial Bodies meet with from hence, is almoſt

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nothing at all. So that the ſaid Bodies may, for all this, continue many Thouſands of Years without any ſenſible Alteration in their Motions.

We ſhall now leave Mr. *Gordon* to his ſecond Thoughts, and give a diſtinct Account of the Foundation, on which is built a ſhort Way of Argumentation often uſed by the Great *Newton* and his Followers, to prove one Proportion to be compounded of other two. In order whereto we ſhall premiſe the following *Propoſition* or *Lemma.*

Proposition XLVIII.

Lem. Fig. 46, 47.

In rectangular Parallelograms R, *r*, *the Proportion of two Sides* A, *a*, *is compounded of the direct Proportion of the Rectangles themſelves* R, *r*, *and the reciprocal Proportion of the other two adjoining Sides* B, *b* : *that is*, $\frac{A}{a}$ is $= \frac{R}{r} \times \frac{b}{B}$.

For

For (by 23. 6. *Eucl.*) $\frac{R}{r}$ is $= \frac{A}{a} \times \frac{B}{b}$. Therefore, if we divide by $\frac{B}{b}$, there will be $\frac{A}{a} (= \frac{R}{r} \div \frac{B}{b}) = \frac{R}{r} \times \frac{b}{B}$. W. W. D.

Proposition XLIX.

Theor. *Fig.* 46, 47.

If there be six Quantities so related to one another, that when the third and fourth are the same or equal, the first is to the second as the fifth to the sixth; and when the fifth and sixth are the same or equal, the first is to the second as the third to the fourth: Then, when neither the third is equal to the fourth, nor the fifth to the sixth, the Proportion of the first to the second is compounded of the Proportions of the third to the fourth, and the fifth to the sixth.

This Theorem in the Newtonian *Stile would be expressed thus.* If three Quantities (*that is* really, three Proportions of six Quantities) be so qualified, that, the

second being given (*that is* really, the second Proportion being a Proportion of Equality) the first is directly as the third; and the third being given, the first is directly as the second: Then, neither the second nor the third being given, the first is as the second and third conjunctly, or directly as the second and directly as the third.

The said Theorem, in the first Expression, is demonstrated thus. The first and second Quantities will be represented by two Rectangles R and *r*, the third and fourth by the Bases B and *b*, and the fifth and sixth by the Altitudes A and *a*: Because (by *Sch.* 1. 6. *Eucl.*) when B is = *b*, there is as R : *r* : : A : *a*; and (by 1. 6. *Eucl.*) when A is = *a*, there is as R : *r* : : B : *b*. Therefore then, since the said six Quantities in order are just so related as, and represented by R, *r*, B, *b*, A, *a*; and also since (by 23. 6. *Eucl.*) the Proportion of R to *r* is compounded of the Proportions of B to *b* and A to *a*; it is evident that the Proportion of the first of the foresaid Quantities to the second, is

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compounded of the Proportions of the third to the fourth and the fifth to the sixth. W. W. D.

Proposition L.

Theor. Fig. 46, 47.

I*F there be six Quantities so related, that when the third is equal to the fourth, the first to the second is reciprocally as the sixth to the fifth; and when the fifth is equal to the sixth, the first is to the second directly as the third to the fourth: Then, when neither the third is equal to the fourth, nor the fifth to the sixth, the Proportion of the first to the second is compounded of the direct Proportion of the third and fourth, and the reciprocal Proportion of the fifth and sixth.*

This Theorem in the Newtonian *Stile would be expressed thus.* If three Quantities (*that is* really, three Proportions) be so qualified, that, the Second being given (or really, being a Proportion of equality) the first is reciprocally as the

third ; and the third being given, the first is directly as the second: Then, neither the second nor the third being given, the first is directly as the second, and reciprocally as the third.

The said Theorem expressed the first way, is demonstrated thus. The first and second Quantities may be represented by the Altitudes A, *a* of two Rectangles R, *r*, the third and fourth by the Rectangles themselves R, *r*, and the fifth and sixth by the Bases *b*, B, in a contrary Order: Because (by 14. 6. *Eucl.*) when R is = *r*, there is reciprocally as A : *a* :: *b* : B; and (by *Sch.* 1. 6. *Eucl.*) when B is = *b*, there is directly as A : *a* :: R : *r*. Therefore then, since the said six Quantities are just so related as the Altitudes A, *a*, the Rectangles R, *r*, and the Bases B, *b*; and also, since (by 48 *Prop.*) the Proportion of A to *a* is compounded of the direct Proportion of R to *r* and the reciprocal Proportion of *b* to B; it is evident, that the Proportion of the first of the foresaid Quantities to the second, is compounded of the direct Proportion of the third and

fourth,

fourth, and the reciprocal Proportion of the fifth and sixth. W. W. D.

A Scholy.

SUPPOSE now one was to prove, in Sir *Is. Newton*'s short Way, the 6th *Proposition*, *viz.* that *in uniform Motions, the Proportion of the Spaces run through is compounded of the direct Proportions of the Times and Celerities*; which in his concise Stile would be expressed thus, *The Space run through is as the Time and Celerity conjunctly.*

His Proof is expressed thus. The Time being given, the Space is directly as the Celerity; and the Celerity being given, the Space is directly as the Time: Wherefore, neither the Time nor the Celerity being given, the Space is as the Time and Celerity conjunctly. The true Meaning is; the Times being the same or equal, the Spaces are (by 1 *Prop.*) directly as the Celerities; and the Celerities being the same or equal, the Spaces are (by 2 *Prop.*) directly as the Times:

Wherefore, neither the Times being the ſame or equal, nor yet the Celerities, the Proportion of the Spaces is, compounded of the direct Proportions of the Times and Celerities. Now that this is a true and concluſive Argument and Way of Reaſoning, is evident from 49 *Prop.* becauſe the two Spaces, the two Times, and the two Celerities here, are ſix Quantities juſt ſo related to one another, as thoſe ſix in that *Prop.*

Suppoſe again one was to demonſtrate in *Newton's* conciſe Method the 1 *Cor.* of 6 *Prop.* viz. that *in uniform Motions, the Proportion of the Times is compounded of the direct Proportion of the Spaces, and the reciprocal Proportion of the Celerities*; which in his brief Stile would be expreſſed thus, *The Time is directly as the Space, and reciprocally as the Celerity.*

His ſhort Proof would be expreſſed thus: The Space being given, the Time is reciprocally as the Celerity; and the Celerity being given, the Time is directly as the Space: Wherefore, neither the Space nor the Celerity being given, the

Time

Time is directly as the Space, and reciprocally as the Celerity. The true Meaning is; the Spaces being the same or equal, the Times are (by 4 *Cor.* 6 *Prop.* which has no Dependence on 1 *Cor.*) reciprocally as the Celerities; and the Celerities being equal, the Times are (by 2 *Prop.*) directly as the Spaces: Wherefore, neither the Spaces nor Celerities being equal, the Proportion of the Times is compounded of the direct Proportion of the Spaces and the reciprocal Proportion of the Celerities. Now that this is a true and conclusive Argument, is evident from 50 *Prop.* because the Times, the Spaces, and the Celerities here, are six Quantities just so related and qualified, as those six in that *Prop.*

We shall conclude this short Treatise, with the Great *Newton*'s own Demonstration of his general Law of centripetal Forces: In order whereto we shall premise the two following *Lemma's.*

Proposition LI.

Lem. Fig. 36.

THE *infinitely little or nascent Lines c f, b r, produc'd or describ'd by a centripetal Force, in equal Particles of Time, are as the centripetal Forces or Impulses in the Points* P, *p; at least infinitely near.*

This, I think, is incontrovertible; because the centripetal Forces in P and *p* are the Causes that produce the nascent Lines *c f* and *b r*. See *Sch.* 44 *Prop.*

Proposition LII.

Lem.

THE *Spaces that a Body, constantly urged by a regular Force, describes, are, in the Beginning of the Motion, in a duplicate Proportion of the Times in which they are described.*

This is *Newton's* 10 *Lem.* 1 *Lib. Princip.* and it is a plain Consequence of *Schol.*

22

22 *Prop. preced.* because a Force that constantly acts regularly, does, in the Beginning of the Motion, act uniformly or equally, at least infinitely near so.

We shall now give *Newton's* own Demonstration of his general Law of centripetal Forces, which, though for Clearness sake we shall enlarge it, I'm confident every Body will grant to be according to his Mind.

Proposition LIII.

Theor. Fig. 36.

EVERY Thing being supposed as in 36 Prop. *I say that the centripetal Force in any Point* P *of the Curve will be reciprocally as the nascent or evanescent Solid* $\frac{SPq \times BDq}{BR}$.

The centripetal Forces and Times being denoted as in 36 *Prop.* if, in the infinitely small or nascent Figures PRBD, *prbd*, the nascent Lines BR, *br* be described in equal Times, they are (by 51 *Prop*) as the centripetal Forces in the Points

Points P, p, or as V, v; and, if the centripetal Forces in P, p be equal, the nascent Lines BR, br are (by 52 *Prop.*) as the Squares of the Times in which they are described, or as T^2, t^2. Therefore, if neither the Times nor the centripetal Forces be equal, the Proportion of BR to br, is (by 49 *Prop.*) compounded of the Proportions of T^2 to t^2 and of V to v, *that is*, $\frac{BR}{br}$ is $= \frac{T^2}{t^2} \times \frac{V}{v}$. Whence $\frac{V}{v}$ is $= \frac{BR}{br} \div \frac{T^2}{t^2} = \frac{BR}{br} \times \frac{t^2}{T^2}$. But (by 32 *Prop.*) $\frac{t}{T}$ is $= \frac{Spb}{SPB} =$ (by 41. 1 *Eucl.*) $\frac{Sp \times bd}{SP \times BD}$, and consequently $\frac{t^2}{T^2} = \frac{Spq \times bdq}{SPq \times BDq}$. Therefore is $\frac{V}{v} = \frac{BR}{br} \times \frac{Spq \times bdq}{SPq \times BDq}$. Whence (as in 36 *Prop.*) V is to v as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$; or the centripetal Force in P is reciprocally as the nascent or evanescent Solid $\frac{SPq \times BDq}{BR}$.

W. W. D.

A Scholy.

THIS *Demonſtration*, though perhaps obſcurer to ſome than that of Dr. *Gregory*'s deliver'd in 36 *Prop.* is yet preferable, being more general, becauſe agreeing to all Sorts of regular Curves that can be deſcribed by a projectile and a centripetal Force: Whereas *Gregory*'s Demonſtration agrees only to thoſe Curves to which equicurve Circles may be deſcribed, as is evident, ſince it depends upon *Cor.* 35 *Prop.*

FINIS.

APPENDIX.

AN APPENDIX.

HAVING finished the former Treatise, we shall here for the Diversion and Exercise of the Minds of Youth, controvert some material Things that have been demonstrated there; and advance some Arguments that seem to overthrow them. And since Mr. *Gordon* is so good at contriving Fallacies of his own, he may, if he pleases, try his Faculty at detecting those of others: But what need I speak so, since 'tis to be supposed that he would be glad, that the Knot I am going to tie could not again be loosed. In order to my present Design, I shall premise the two following *Lemma's*, and afterwards compare them together.

Lem. I. *Fig.* 36.

THE nascent or evanescent Subtenses RB, *fc* of the Angle of Contact in a Curve APB*p*, are in the simple Proportion of their conterminal Arches PB, *P C*.

For RB being an infinitely small or nascent Line, the Point B must be infinitely near the Point P, and consequently PB must be an infinitely small or nascent Arch: Whence the said Arch is infinitely little different from a right Line. Therefore the Triangles RPB, *f* P *c* (the Point *c* being between B and P) are to be considered as rectilinear Triangles; but they are also equiangular, because *fc* and RB are parallel. Therefore (by 4. 6. *Eucl.*) as is RB : *fc* :: PB : P *c*. W. W. D.

This Demonstration seems very plain, natural, and convincing.

Lem. 2. *Fig.* 36.

THE naſcent or evaneſcent Subtenſes R B, *fc* of the Angle of Contact in a Curve, are in the duplicate Proportion of their conterminal Arches P B, P C.

This *Lemma* which indeed contradicts the former, was before deduced as an Inference or *Corollary* from 35 *Prop. preced.* Now we ſhall enquire whether has the Advantage of the other.

The Demonſtration of the firſt *Lemma* ſuppoſes only, that the infinitely little Arch P B is a right Line, which, though it be not accurately ſo, is infinitely near ſo; and this Suppoſition ſeems very natural and allowable. Whence it will (by 4. 6. *Eucl.*) immediately, neceſſarily, and rigorouſly follow, that as R B: *f c* :: P B : P C. The Proof then of the ſaid *Lemma* only ſuppoſes one infinitely ſmall Inaccuracy.

The Demonſtration of the ſecond *Lemma* ſuppoſes ſeveral ſmall Inaccuracies. *Firſt*, The Demonſtration of the firſt Caſe

of 35 *Prop.* whereon the said *Lemma* depends, supposes that the Arches A D, A d (see *Fig.* 33) coincide with their Chords A D, A d; which is not rigorously true, and is the first Inaccuracy of the same Nature with that in the first *Lemma.* From whence, *Secondly*, It will follow that the Arch A D is a right Line, the same with its Chord A D; and that the Arch A d, a Part of the Arch A D, is a Part of the Chord A D: And consequently d must be consider'd as a Point of the Chord A D. But this being supposed; though the Angle A D C (by 31. 3. *Eucl.*) be rigorously a right one, the Angle A d C will not rigorously be a right one; and then it will not rigorously follow that Chord A $d\,q$ is $= bd \times$ A C, or $\frac{\text{Chord A } d\,q}{\text{A C}}$ is $= bd$, which is a necessary Step in the Demonstration of the first *Case* of 35 *Prop.* and consequently the said *Case* it self will not rigorously follow. This is a second Inaccuracy. Therefore, since there are two Inaccuracies in the Demonstration of the said first *Case*, there must also be two in the second *Lemma* that depends upon it.

But,

But, *Thirdly*, There is yet another Inaccuracy, perhaps as great as, if not greater than either of the other two, in the Demonſtration of *Cor.* 35. *Prop.* which *Corollary* is the ſecond *Lemma* we are juſt now ſpeaking of, in accommodating the Curve to the Circle: This, I believe, will be very obvious to any Body that attentively conſiders the ſaid *Corollary*.

From all which it appears very plain, that the firſt *Lemma* has ſo far the Advantage of the ſecond, that the firſt is to be admitted for Truth, and the ſecond rejected. But I have yet ſomething farther to ſay in behalf of the firſt *Lemma* and againſt the ſecond, which is this. Mr. *Gordon*'s *Theorem* about the Fall of a Body, deſcribing a Curve, from the Tangents, which I have given at the Beginning of *Scholy* 44 *Prop.* looks really ſo like a Paradox as to be utterly incredible: Yet if the ſecond *Lemma* be admitted for Truth, that *Theorem* is really and truly demonſtrable from it; ſee the juſt now mentioned *Scholy*. But if the firſt *Lemma*

be granted to be true, the said *Theorem* will easily be overthrown, and that which appears to be the Truth, proved, *viz.* That, *if* (in *Fig.* 43) *a d be an infinitely small Arch of a Curve, and Tangents be drawn at all the Points of Division of the said Arch divided into an Infinity of Parts; a Body describing the said Arch will fall from the Tangents certain Spaces, and the Sum of all these Spaces will be (infinitely near) equal to the versed Sine* kd *of that Arch.* Which is thus demonstrated.

Suppose the infinitely small or nascent Arch ad bisected in c; then (by 1 *Lem.*) as is $ec : kd :: ac : ad :: 1 : 2$, therefore is $ec = \frac{1}{2}\, kd$. When the Body has come from a to c, the centripetal Force tending to s, has made it fall, from the Tangent of the Point a, the Space ec; and when it is come to d, the said Force has made it fall from the Tangent of c, another Space equal to ec, at least infinitely near so; the Sum of which two Spaces, *viz.* $2\,ec$ is $= kd$, because (as before) ec is $= \frac{1}{2}\, kd$. If again we suppose

pose the Arch ad divided into three equal Parts ab, bc, cd; the Body, by the Force tending to s, will fall from the Tangent of a the Space fb, from the Tangent of b as much, and from the Tangent of c also as much, *that is*, it will fall from the three Tangents of a, b, c, three Spaces, every one of which is equal to fb, the Sum whereof, *viz.* 3 fb is $=kd$, because (by 1 *Lem.*) as is $fb : kd :: ab : ad :: 1 : 3$. In like Manner, if we suppose the Arch ad divided into four equal Parts, the Sum of the Spaces fallen from the Tangents of the Point a and the next three Points of Division, will be equal to kd: And so forth; be the Points of Division ever so many. Therefore the Sum of the Spaces fallen from all the Tangents, while the Body is describing the infinitely little Arch ad, is (at least infinitely near) equal to the versed Sine kd of that Arch. W. W. D.

The first *Lemma* being now admitted for Truth, the general Law of centripetal Forces will be found very different from that of the Great *Newton* formerly de-